Transformers

Feb 24, 2026

*Acknowledgment: Figures based on materials by Dr. Kilho Shin.

Outline

1 Attention

2 Transformer

3 Preview

4 Next Week & Upcoming Assignment

Outline

1 Attention

2 Transformer

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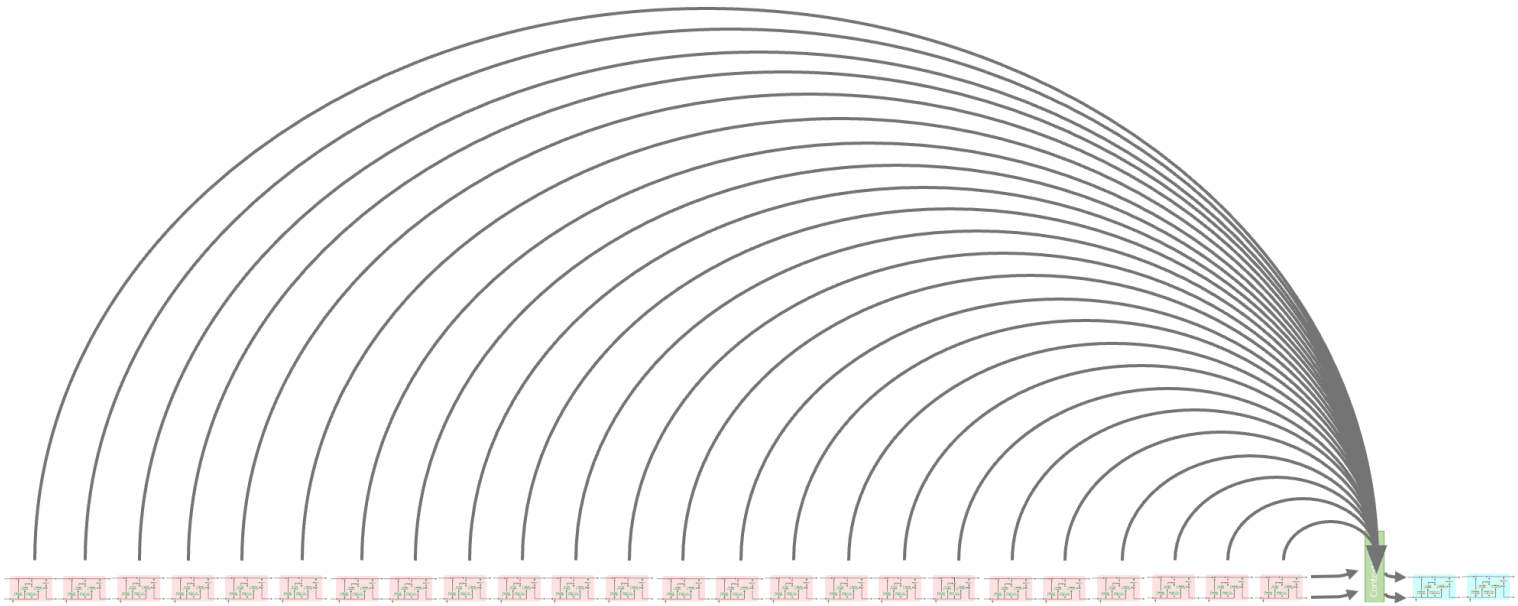
4 Next Week & Upcoming Assignment

Seq2Seq

- What is Seq2Seq?
- Input Sequence $\rightarrow$ Encoder $\rightarrow$ Hidden Representation $\rightarrow$ Decoder $\rightarrow$ Output Sequence

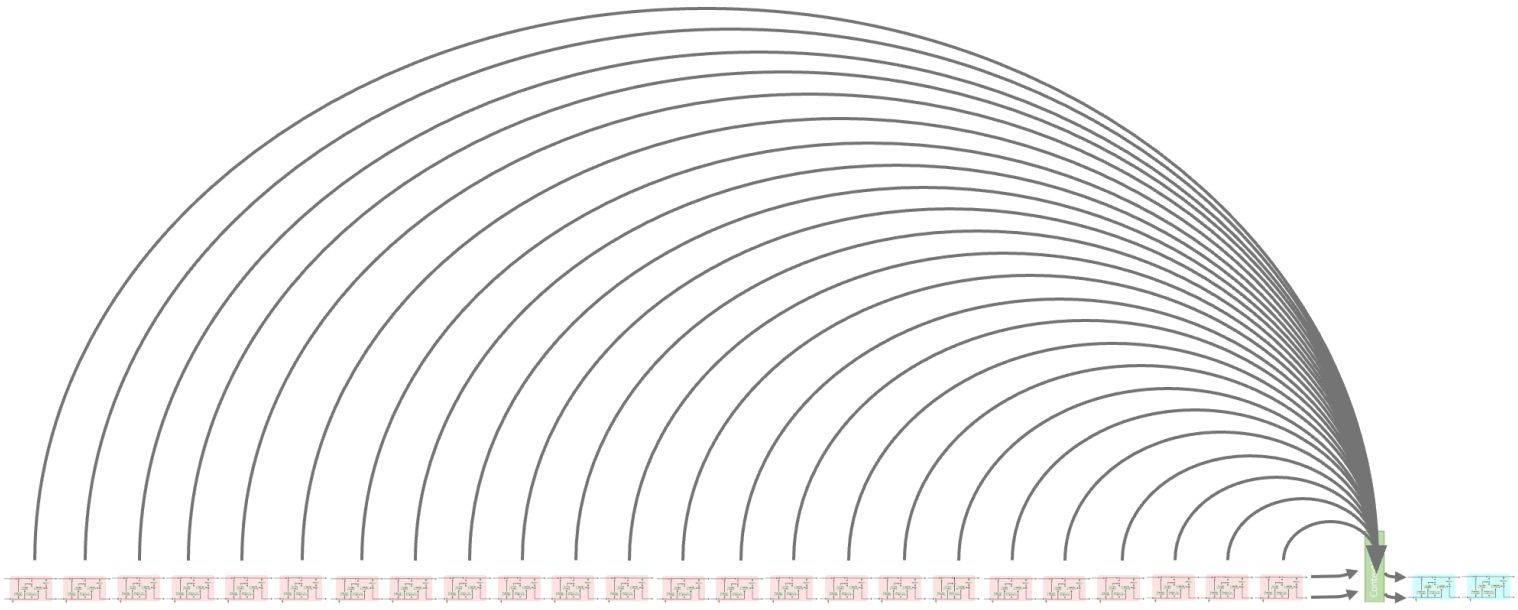
Seq2Seq: Problem

It becomes quite difficult to pack all the information of the input sequence into a fixed-length context vector (*Bottleneck* problem)



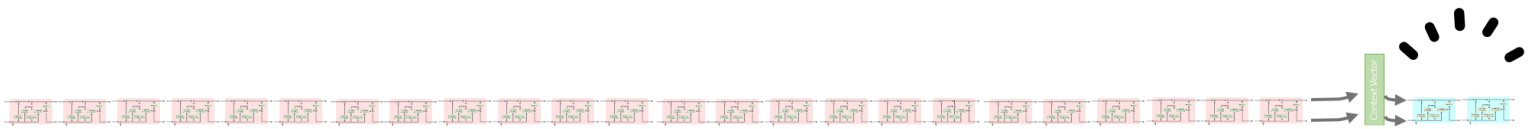
Seq2Seq: Problem

The **attention** mechanism was introduced to address this problem.



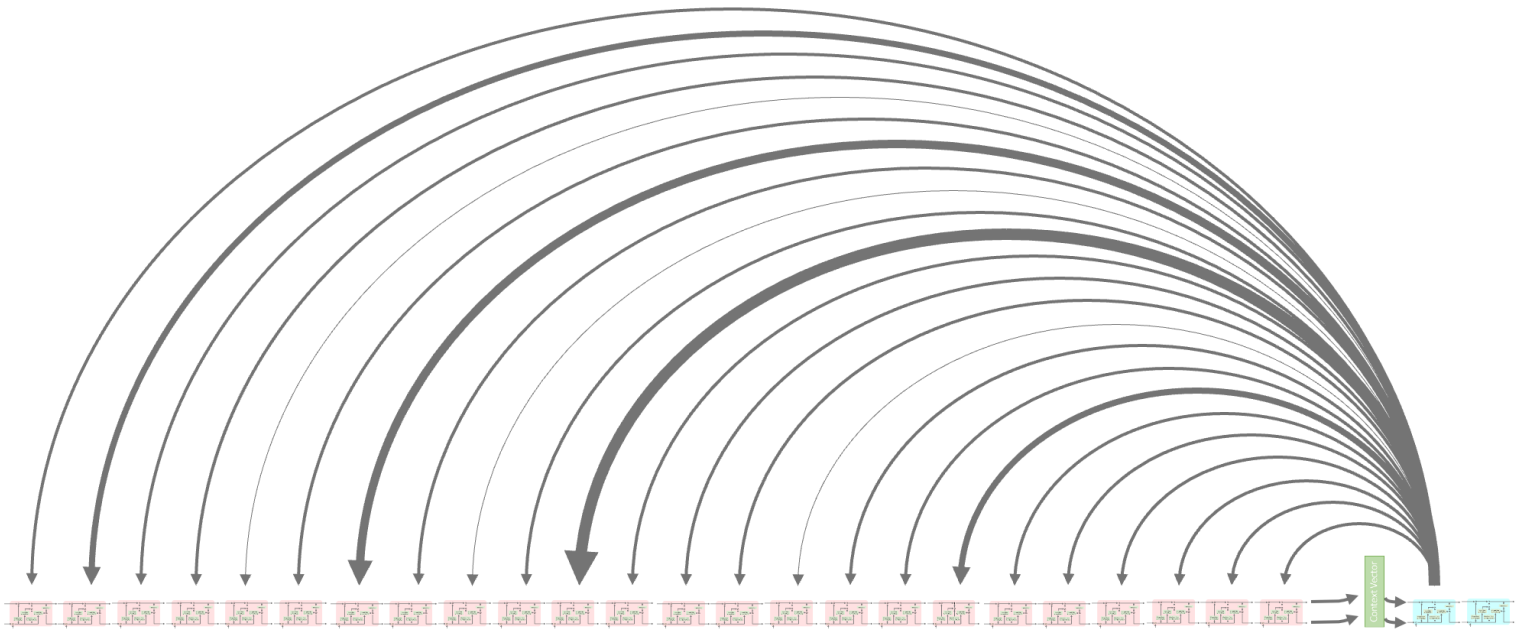
Seq2Seq: Problem

When the decoder generates each word in the output sequence, the attention mechanism



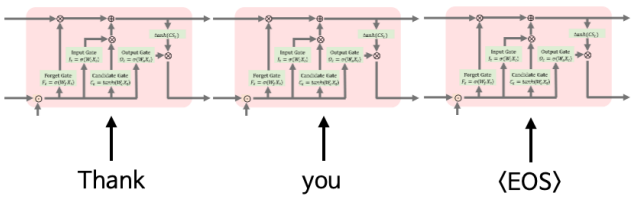
Seq2Seq: Problem

is an algorithm that makes it "attend" to which parts of the input sequence are important.



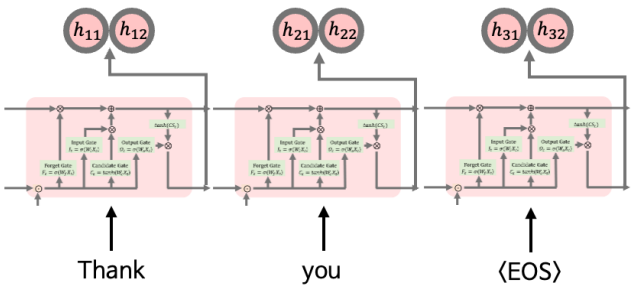
Attention

Suppose the input sequence comes in like this:



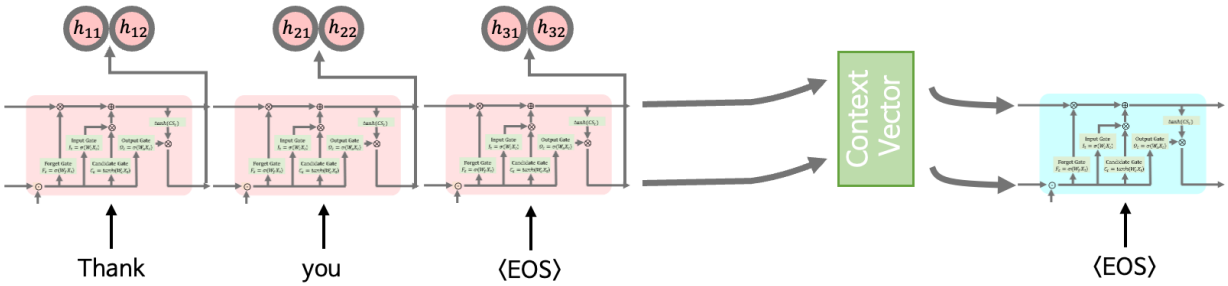
Attention

We store the hidden state for each input word separately.



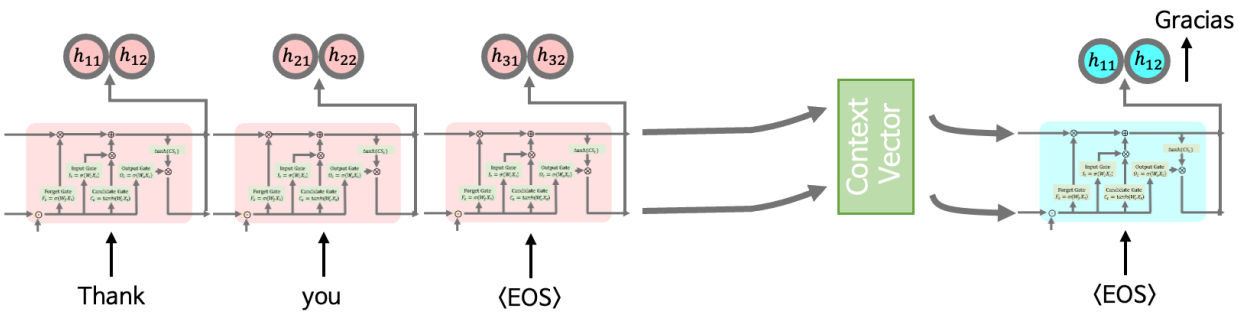
Attention

We build a context vector and feed it to the decoder,



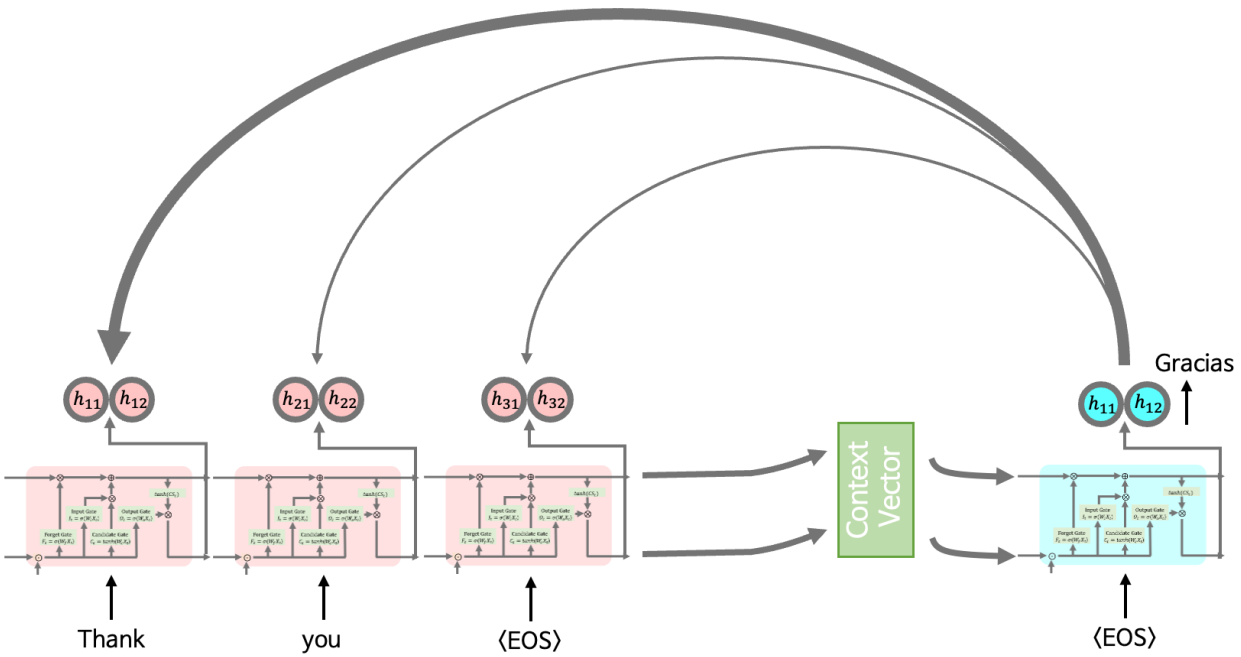
Attention

and obtain the decoder's hidden state and output as follows.



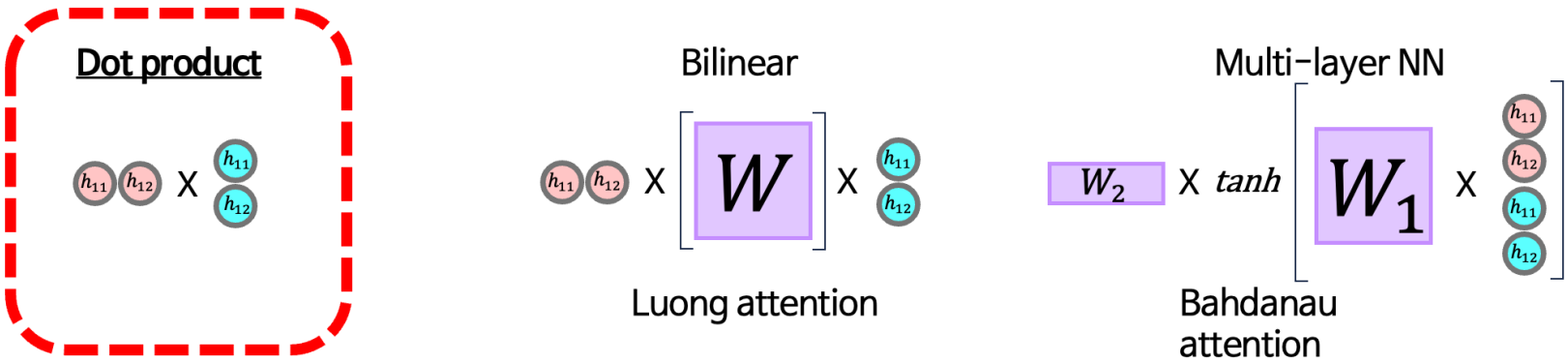
Attention

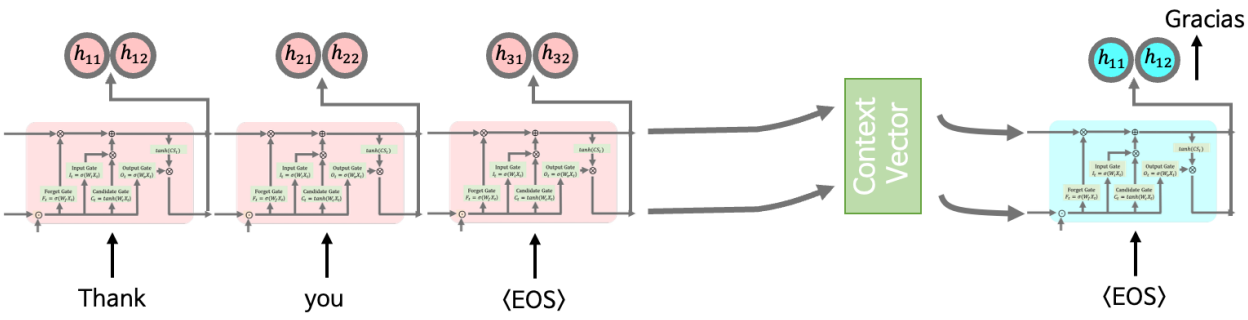
The **similarity** between two vectors shows the relationship between two states.



Attention

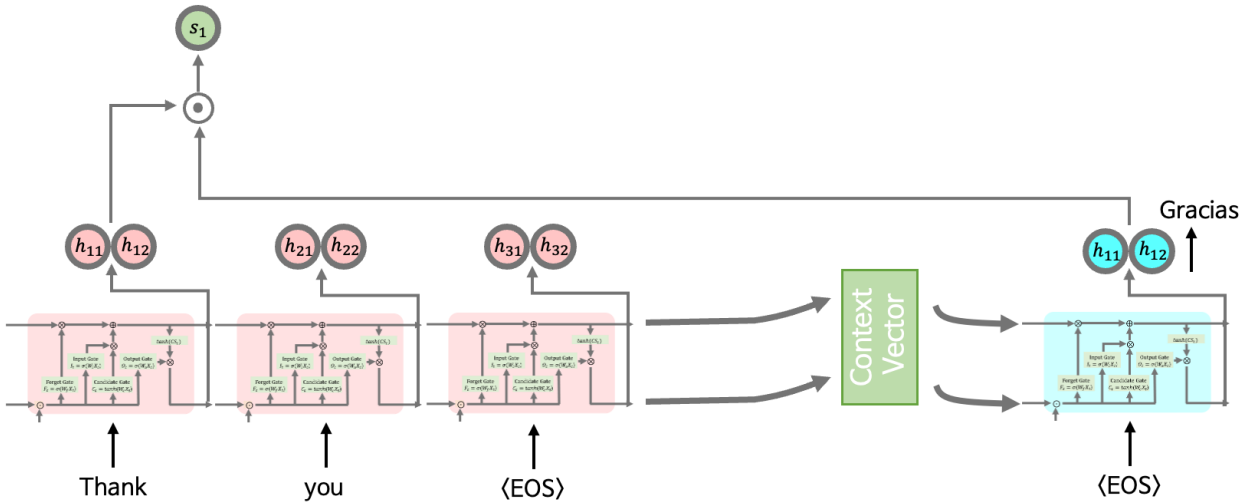
To calculate this, we use the simplest method: the **dot product**.





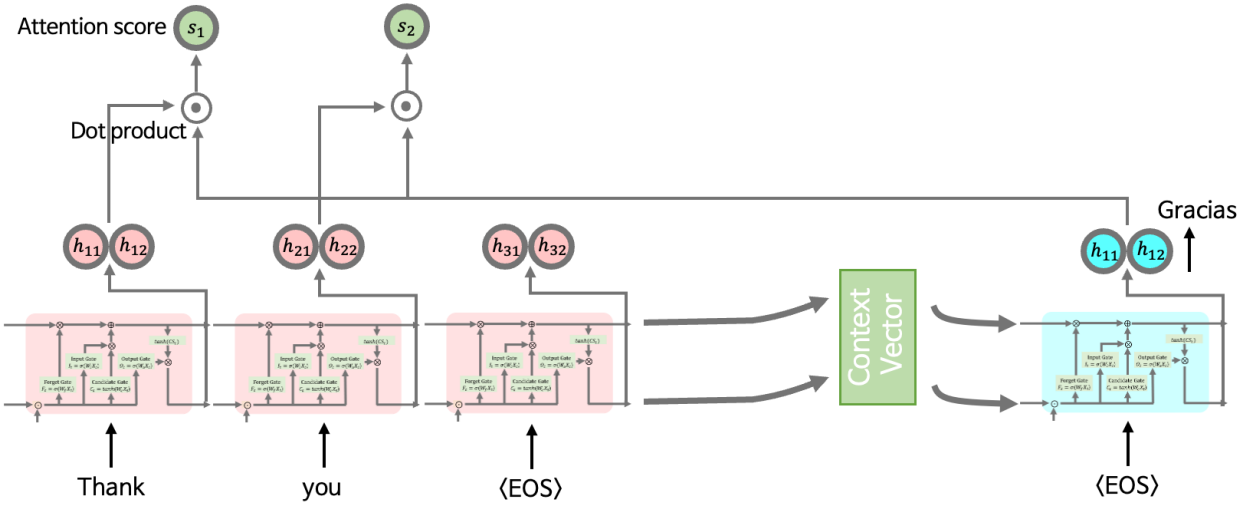
Attention

For example (hidden state 1)



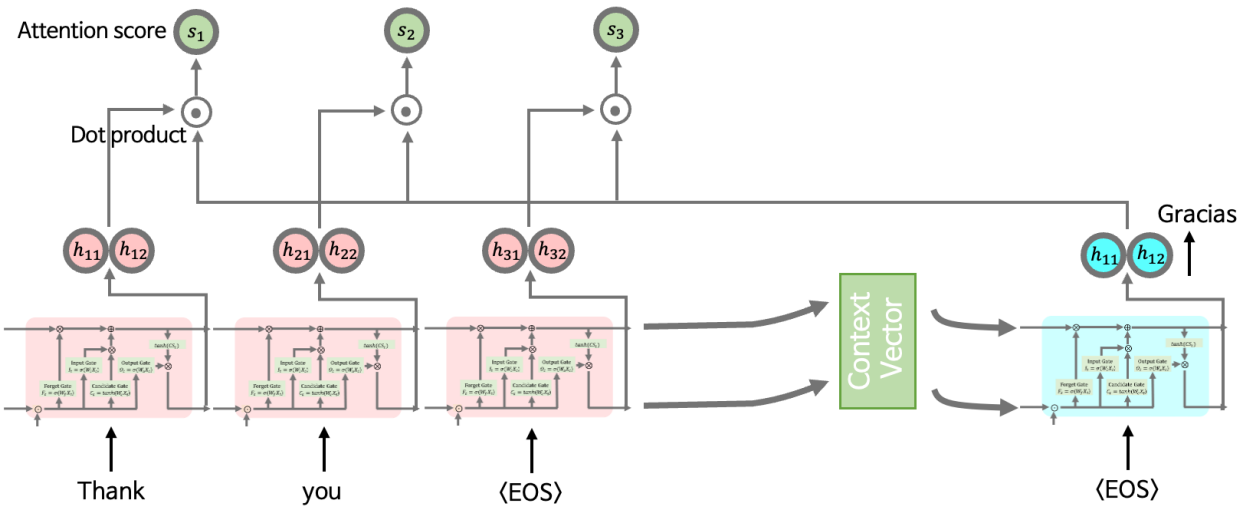
Attention

For example (hidden state 2)



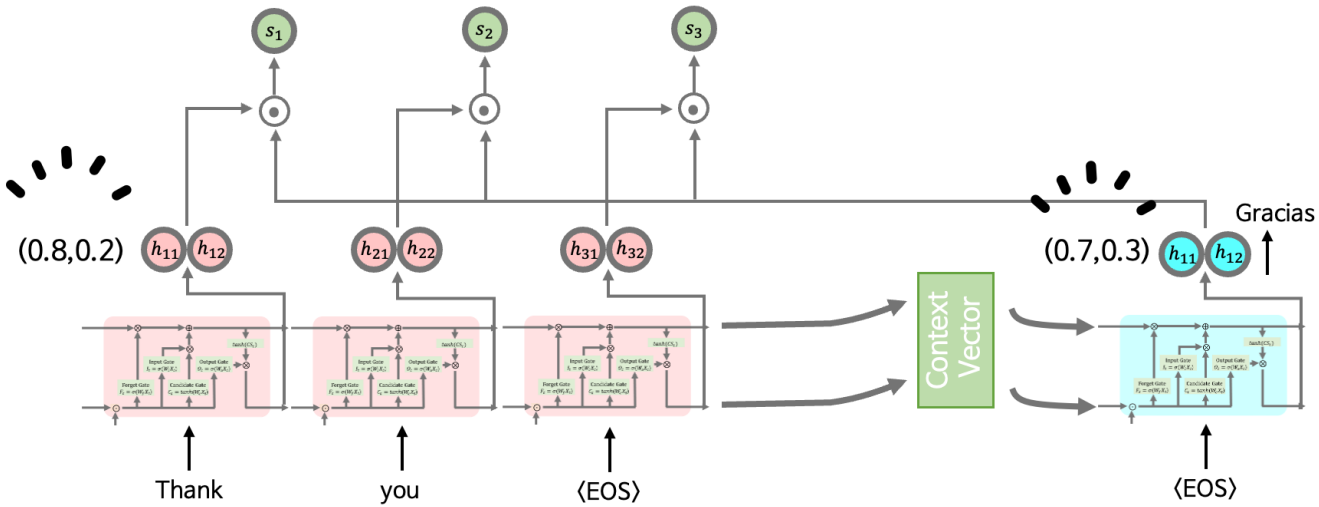
Attention

For example (hidden state 3)



Attention

Suppose the encoder hidden state is (0.8, 0.2) and the decoder hidden state is (0.7, 0.3).



Attention

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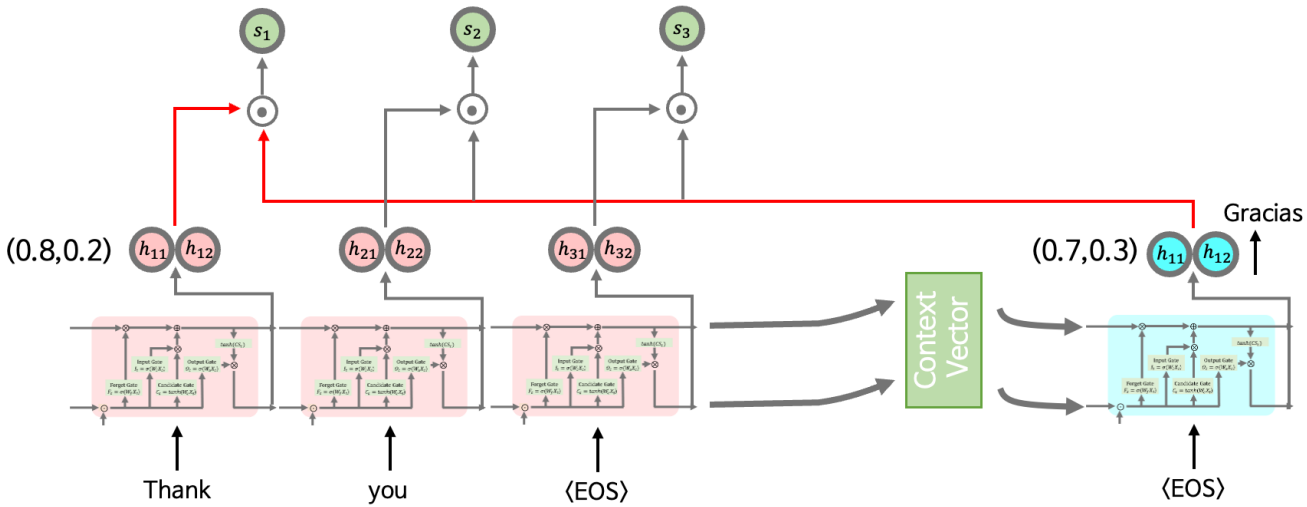
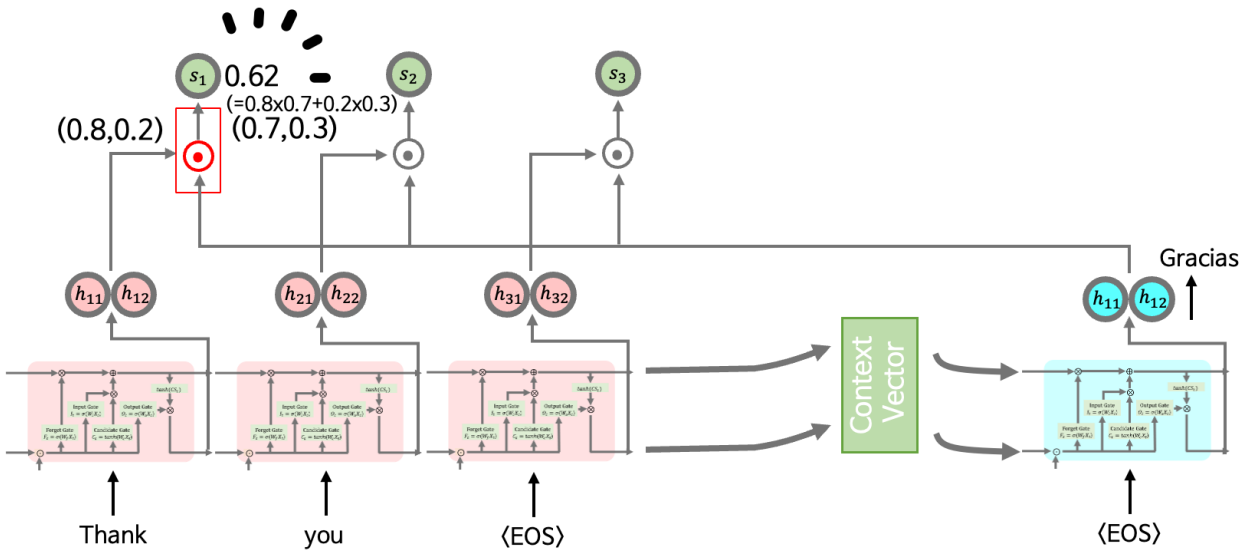


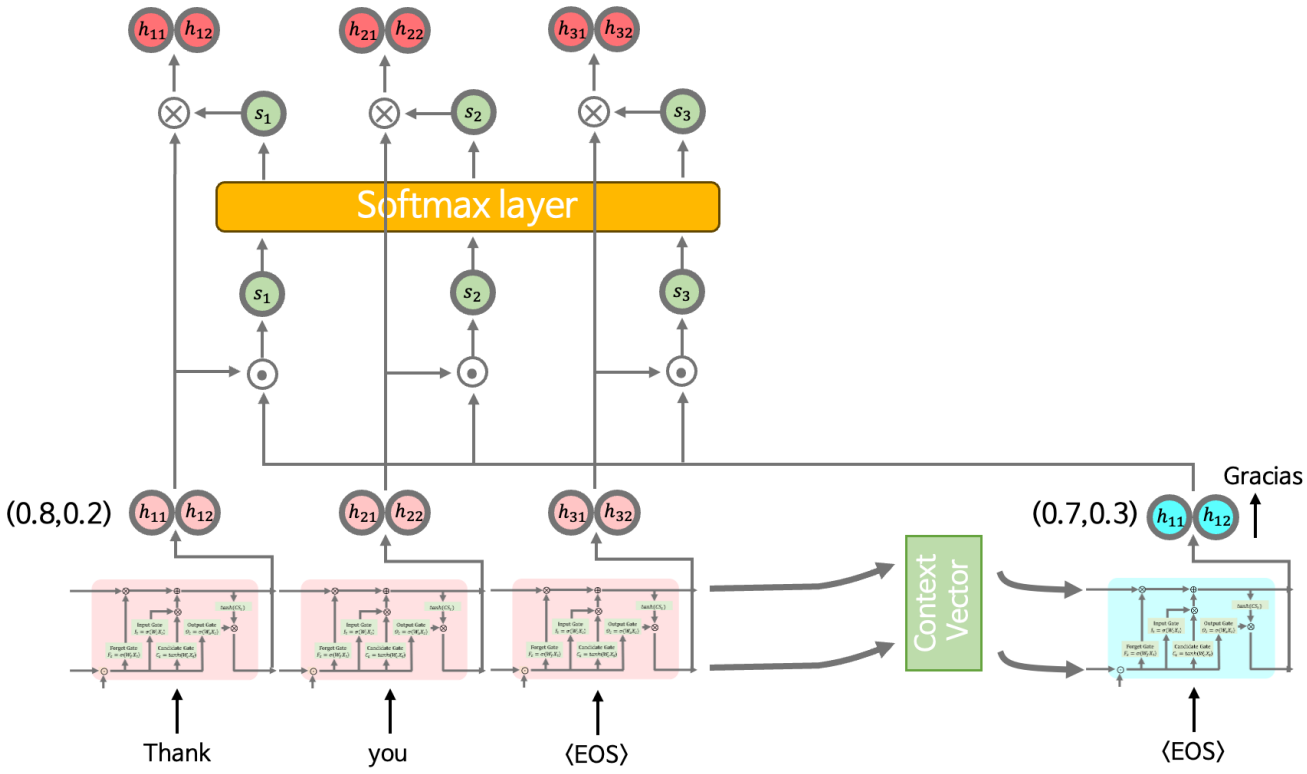
Attention

Then the attention score s1 becomes 0.62 through the following dot-product computation.



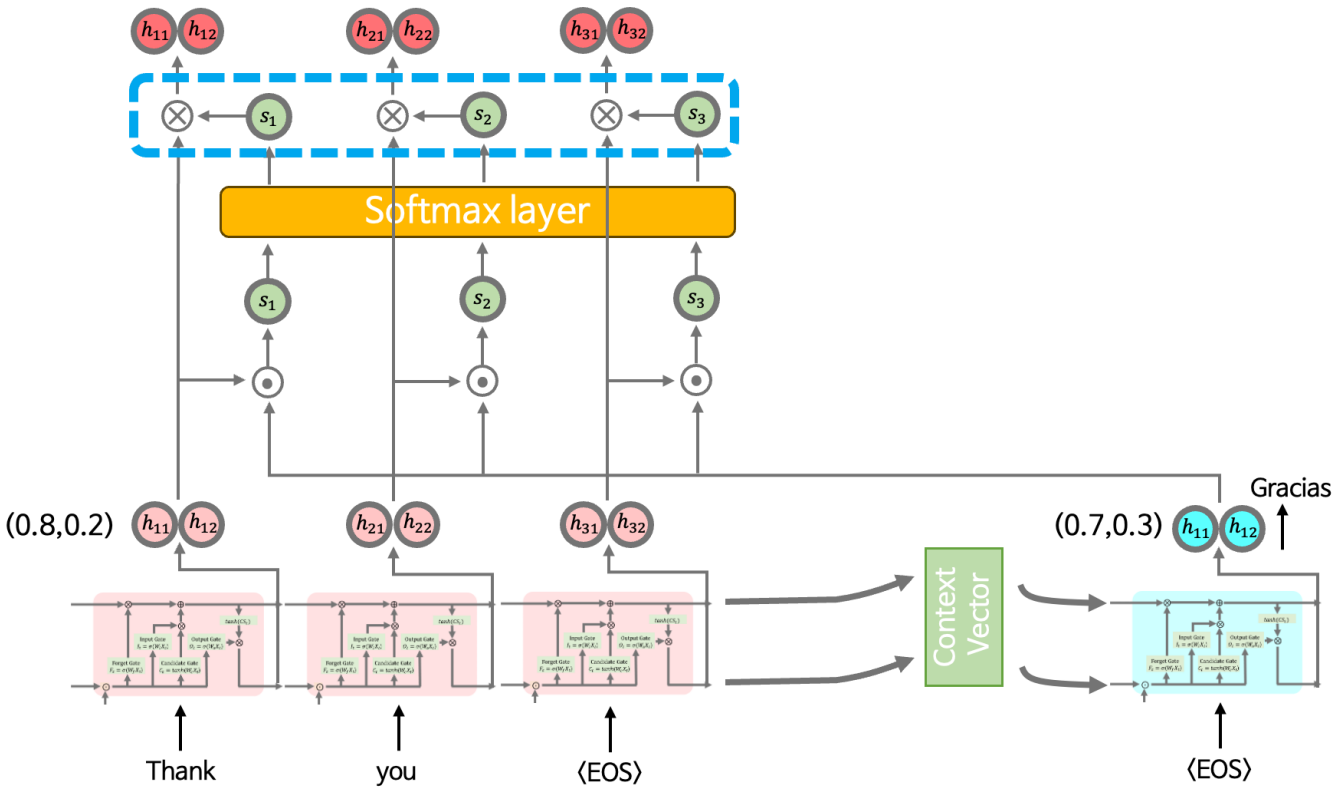
Attention

Next, we compute the **softmax** of each attention score



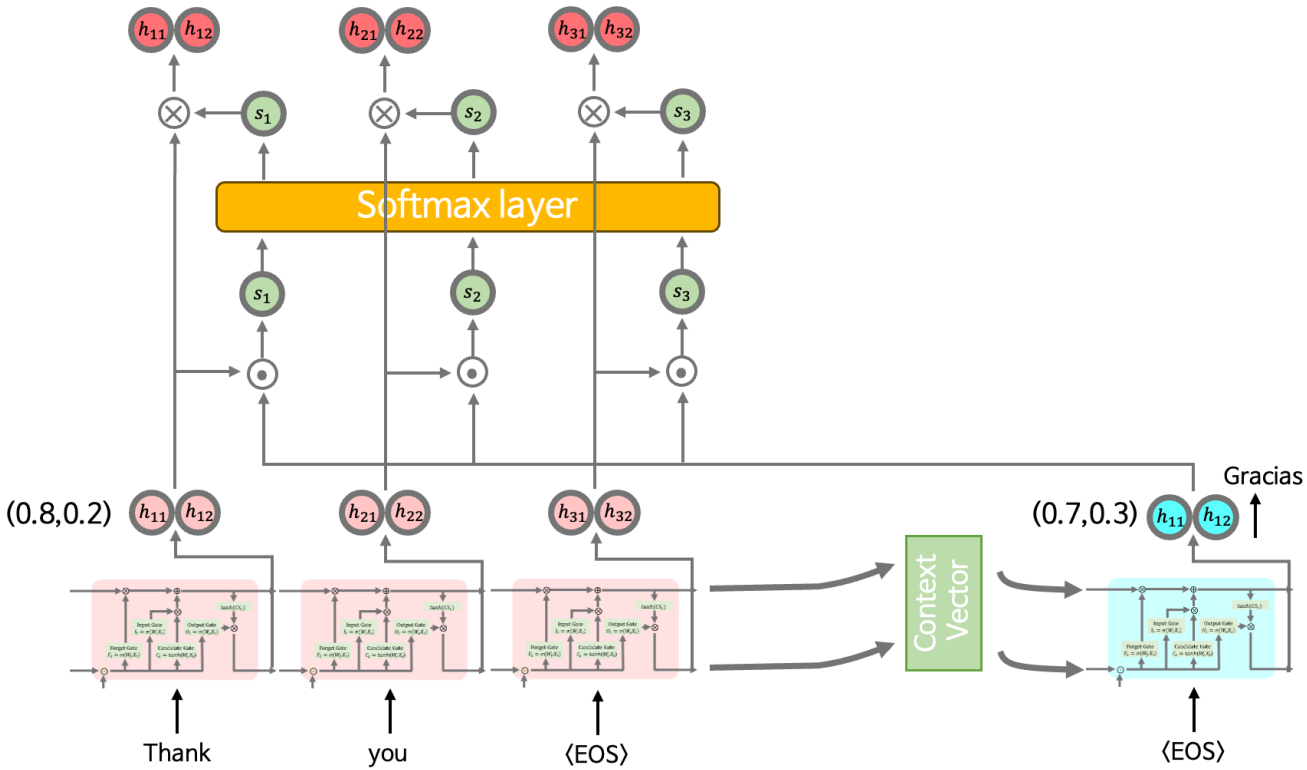
Attention

to convert the attention scores into a **probability distribution** and **normalize** them.



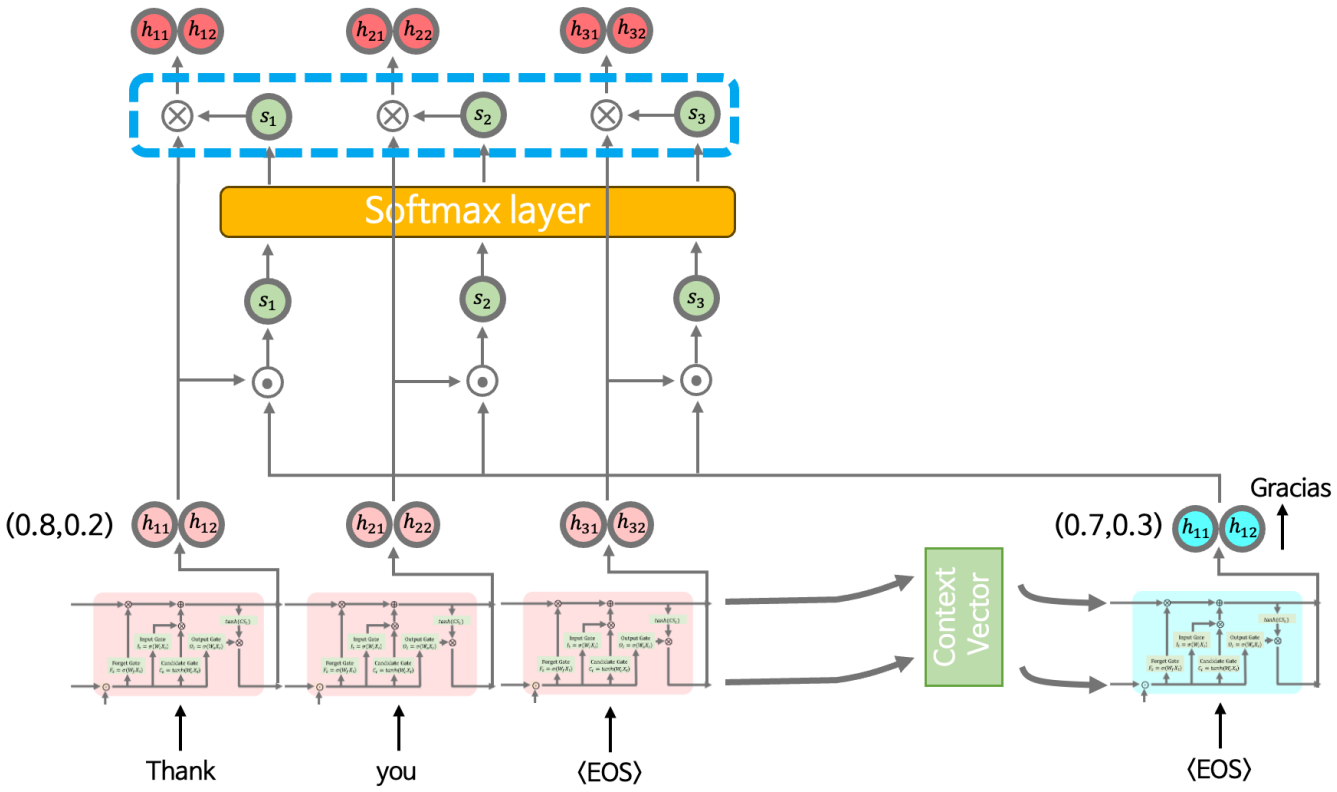
Attention

Next, we multiply each attention score by its corresponding input hidden state.



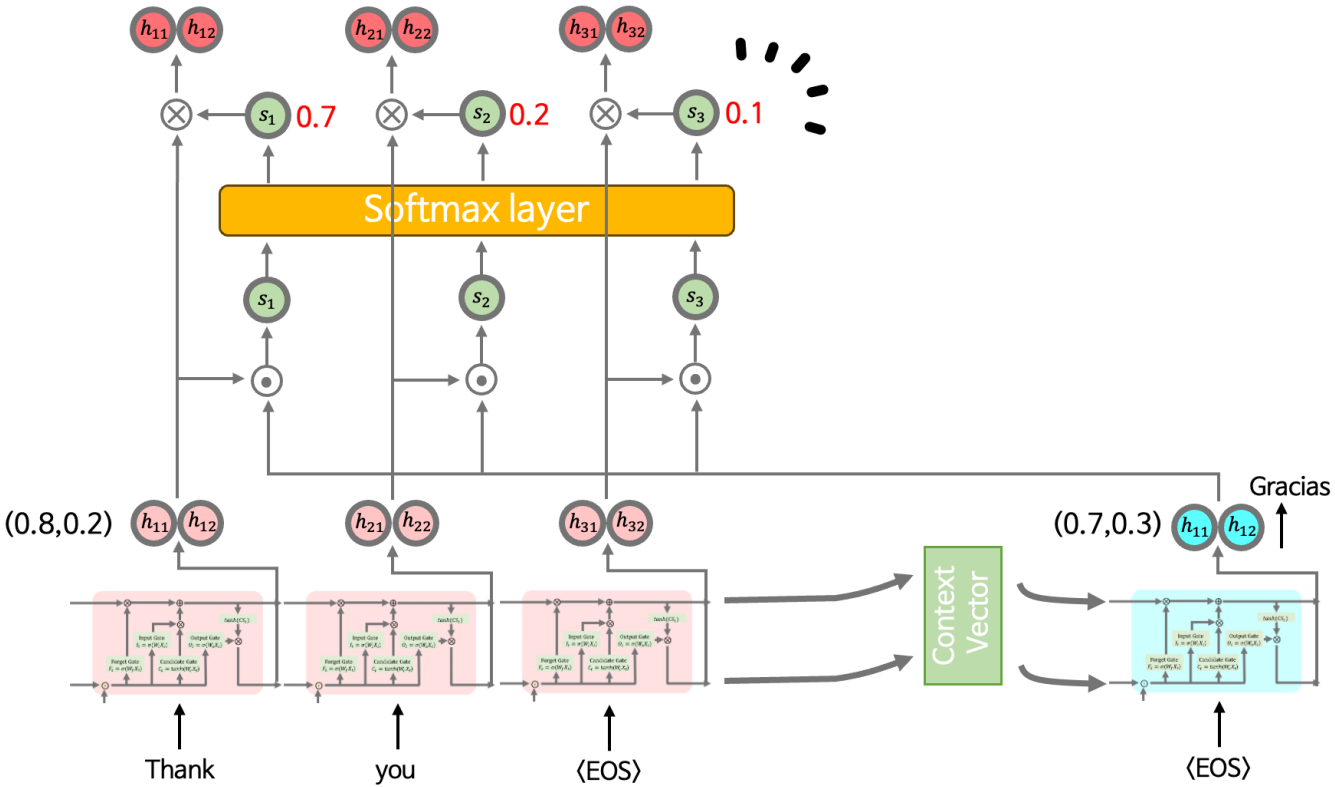
Attention

The multiplication has the effect of **amplifying** the input hidden states according to their attention weights.



Attention

For example, if the attention scores after the softmax layer are as follows—



Attention

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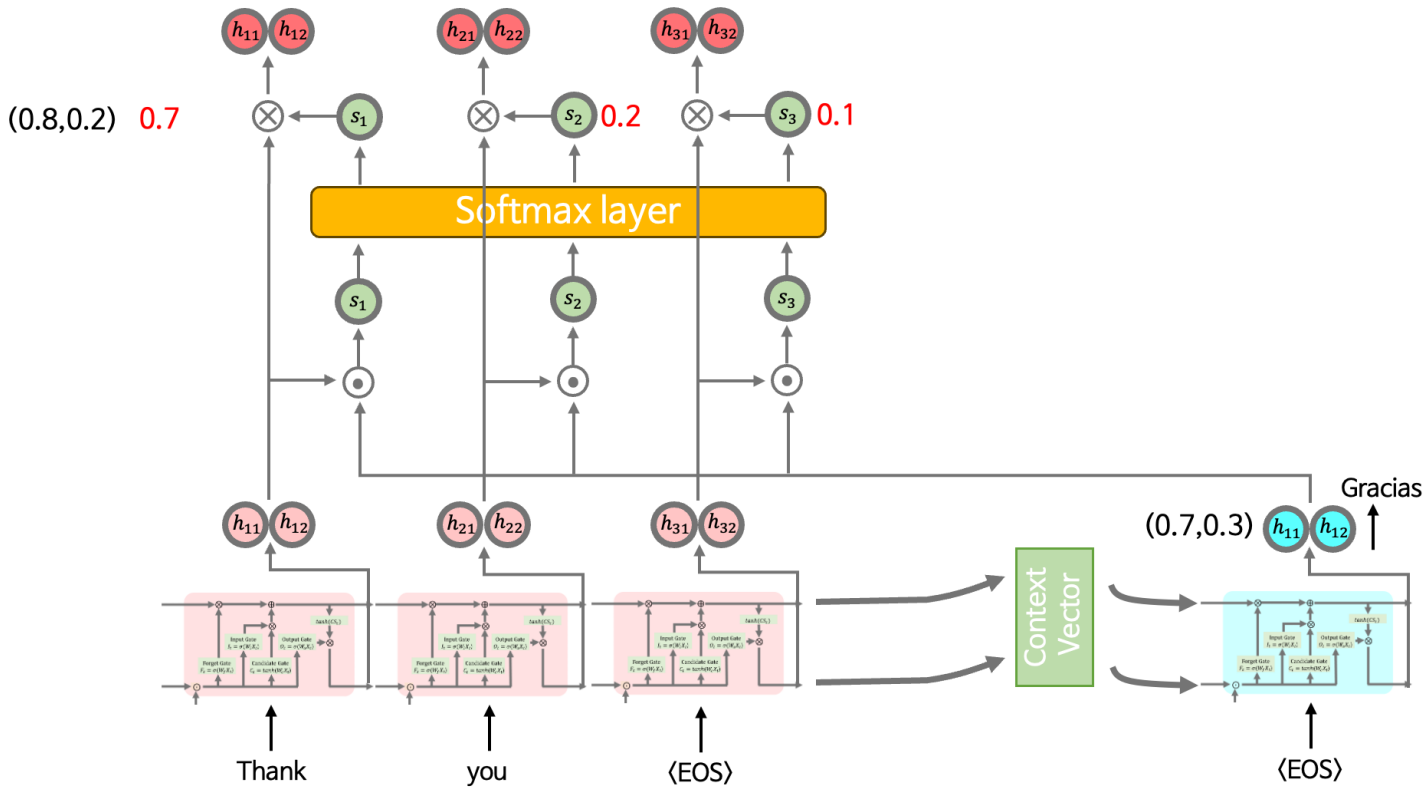
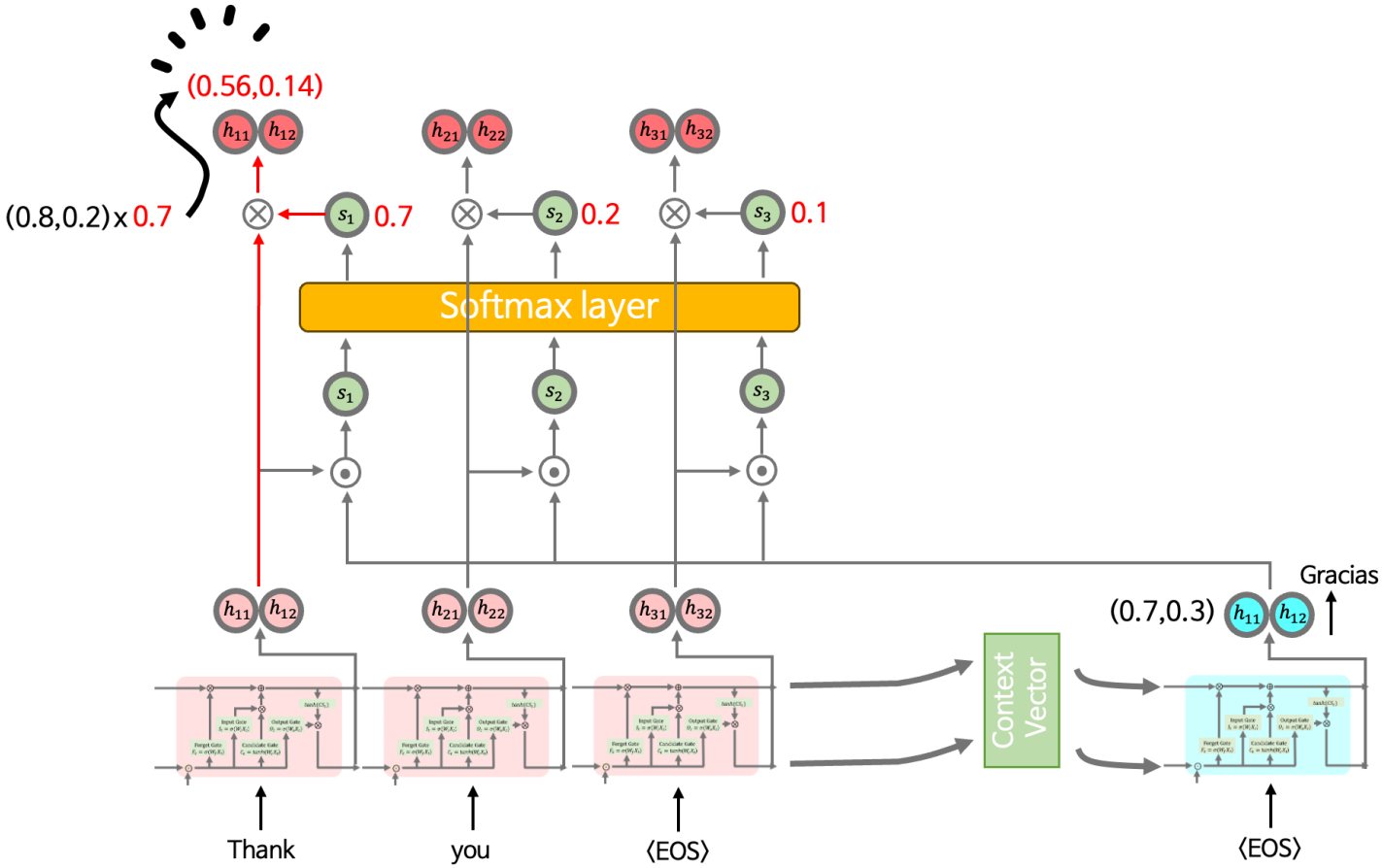


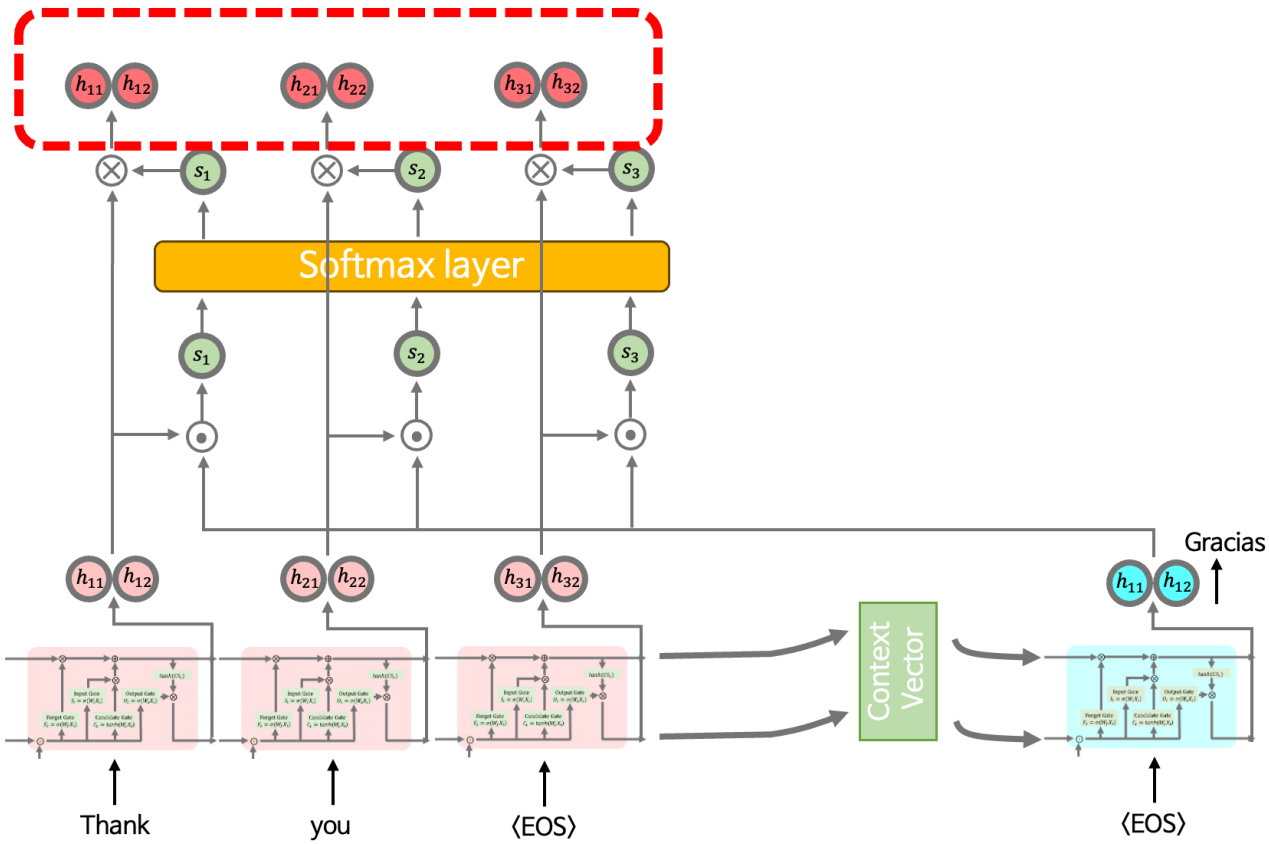
Attention

Then the input state (0.8, 0.2) multiplied by 0.7 becomes (0.56, 0.14).



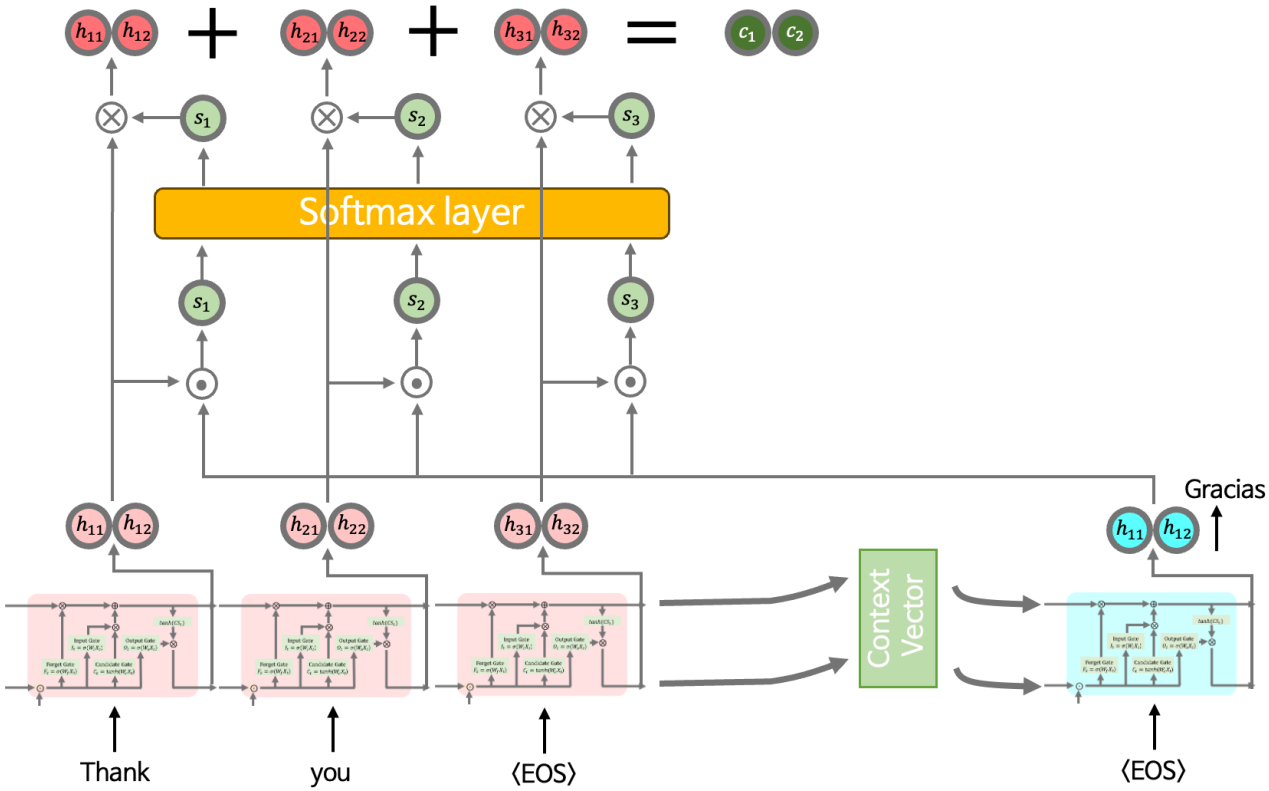
Attention

Now, take these **attention-weighted** input hidden states,



Attention

sum them up, and you obtain a **new context vector**, which integrates information from all words, with more important words contributing proportionally more to the final representation.



Attention is a *general* deep learning technique

- Attention has become the powerful, flexible, general way pointer and memory manipulation in deep learning models. (A new idea from 2010).

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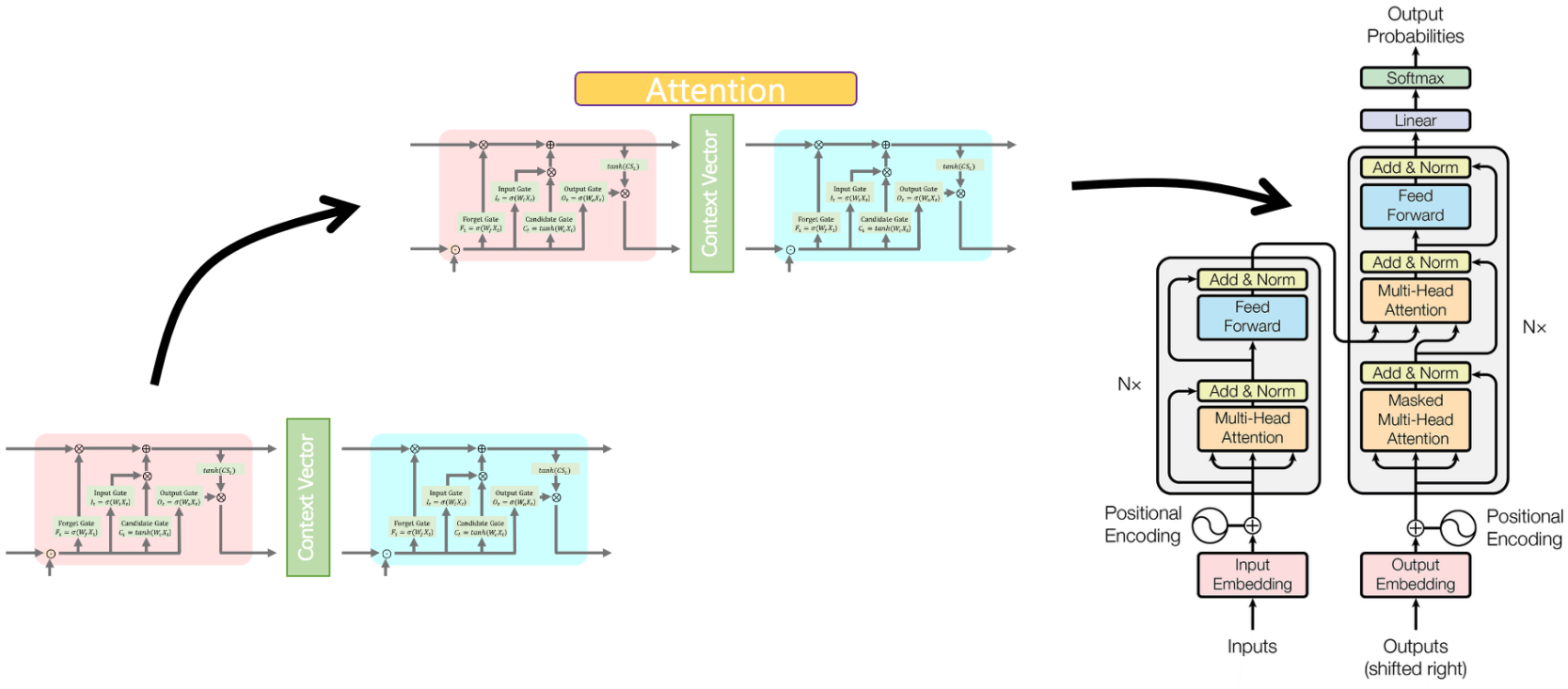
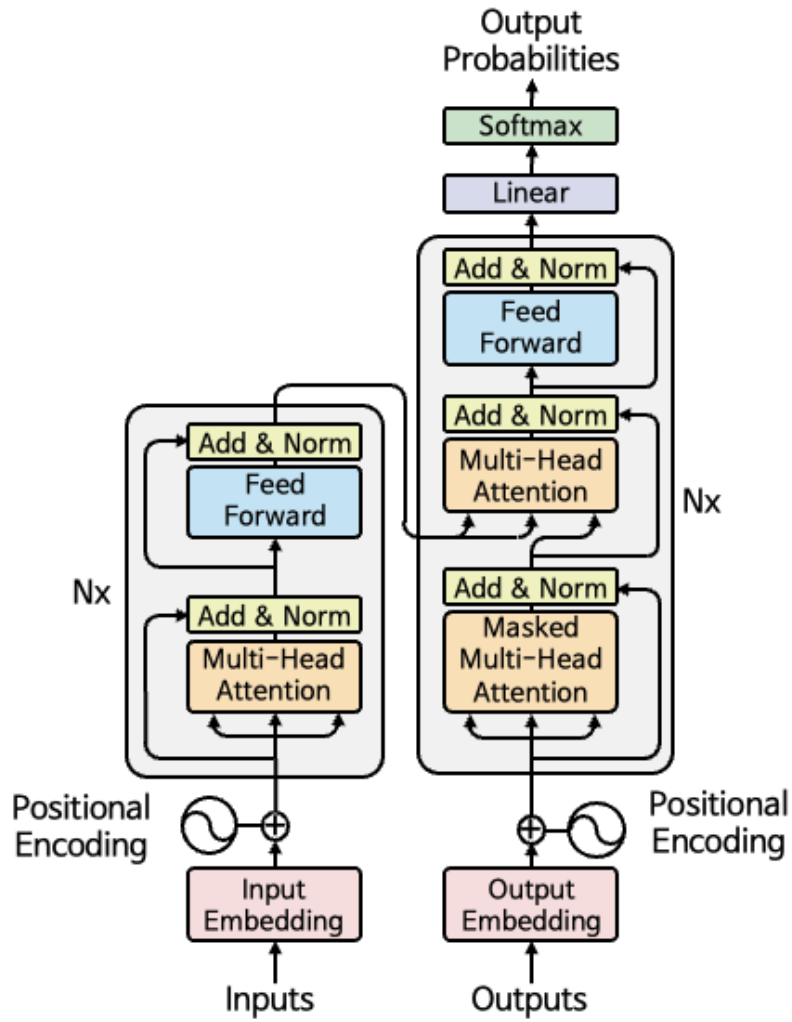
Progress

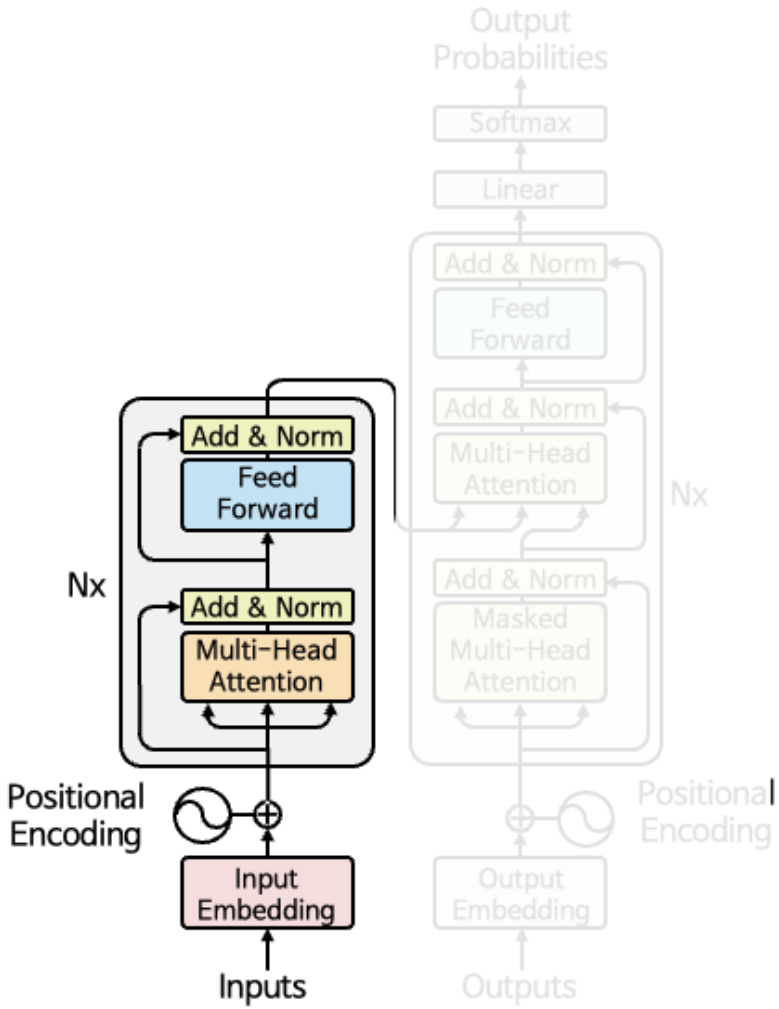
Eventually led to the development of the **Transformer** model.



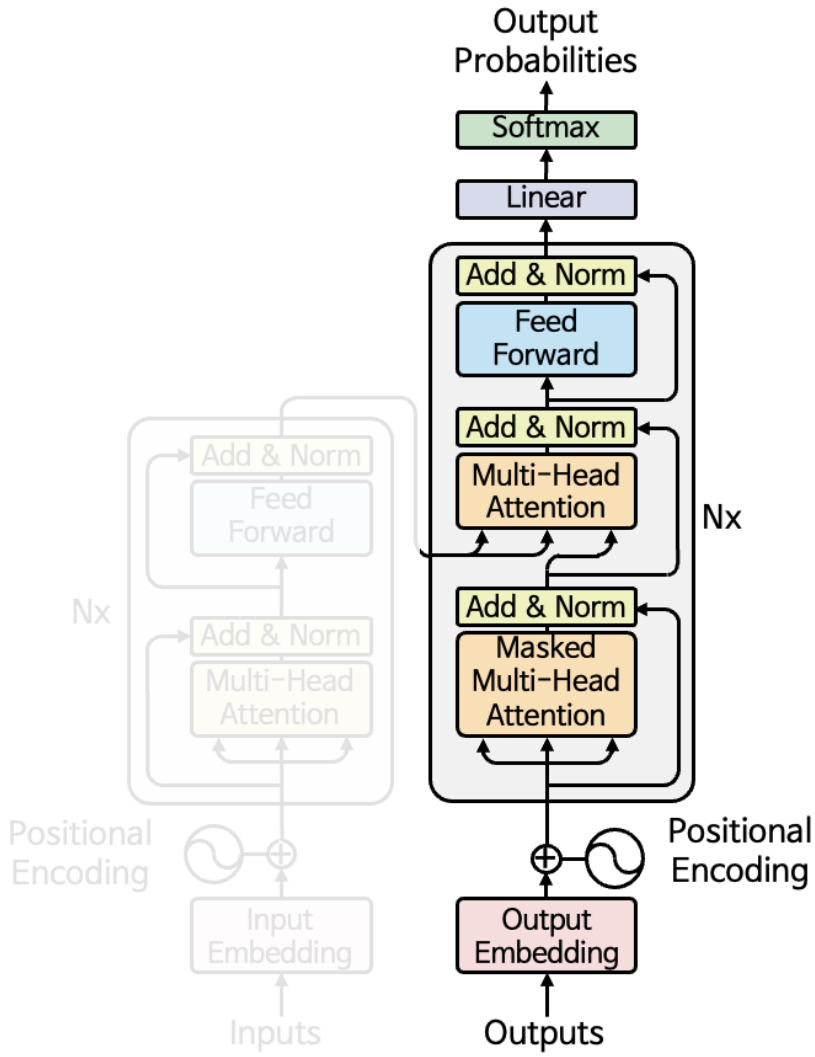
The structure of the Transformer (Vaswani et al., 2017) looks like this:



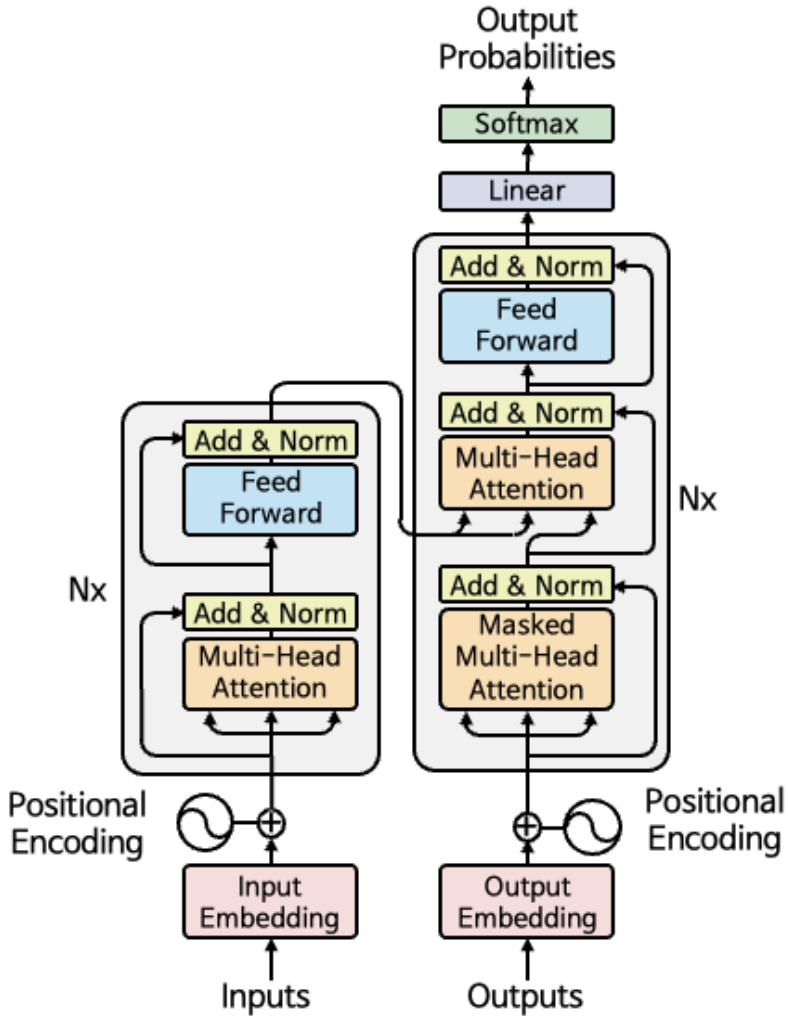
Here we see the **encoder**,



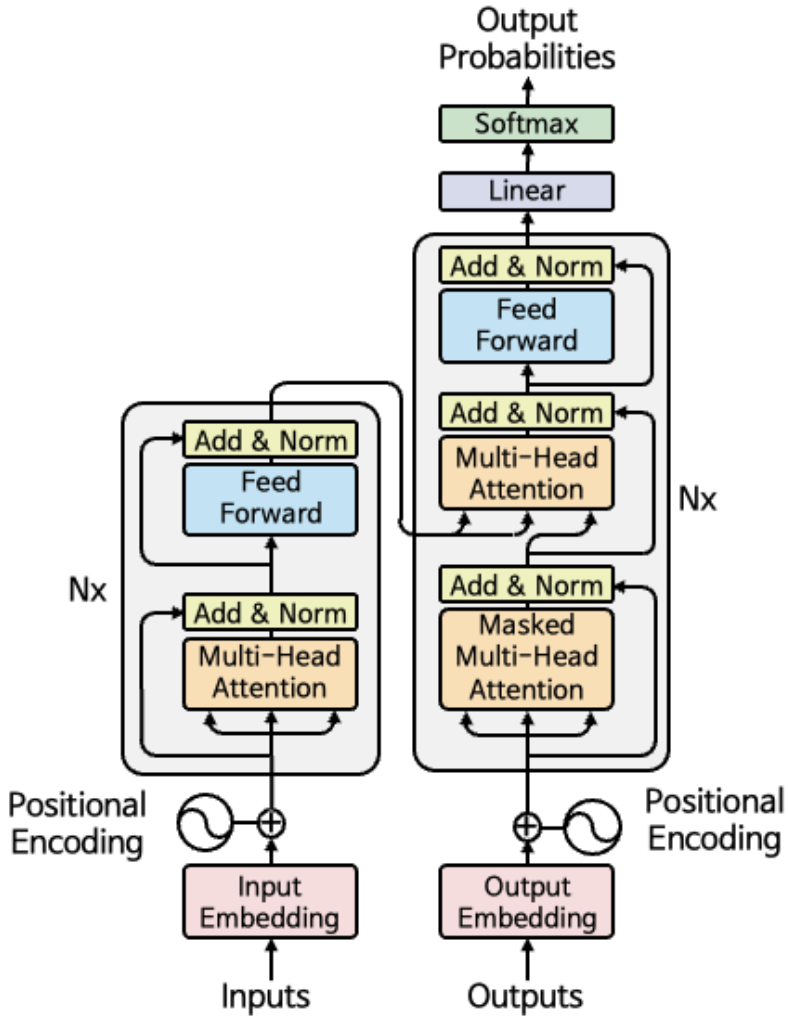
and the **decoder**.



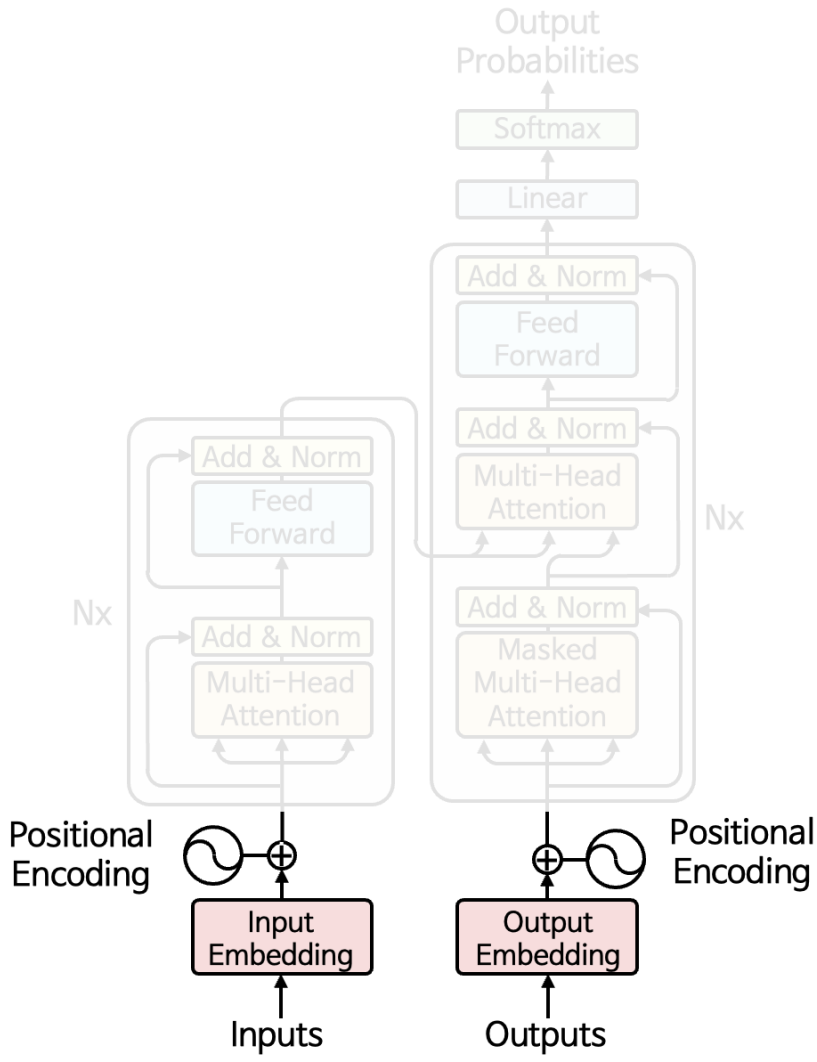
If we look closely at the entire Transformer model,



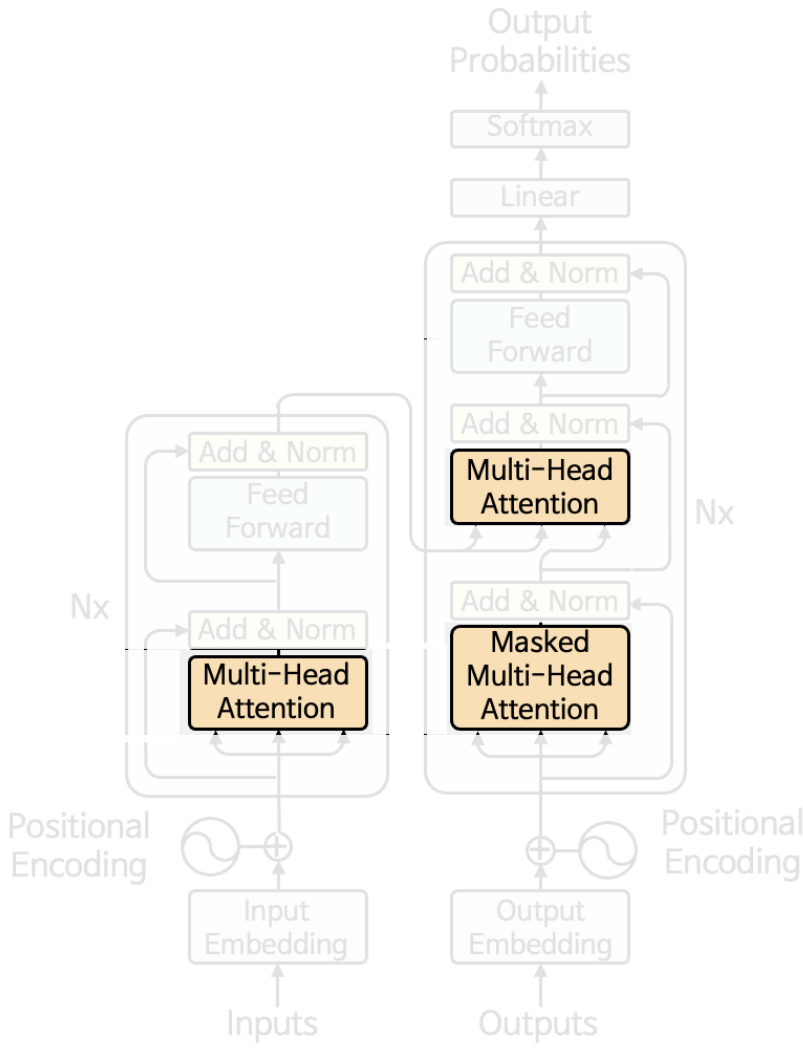
we can see that the same kinds of blocks are **repeatedly stacked**.



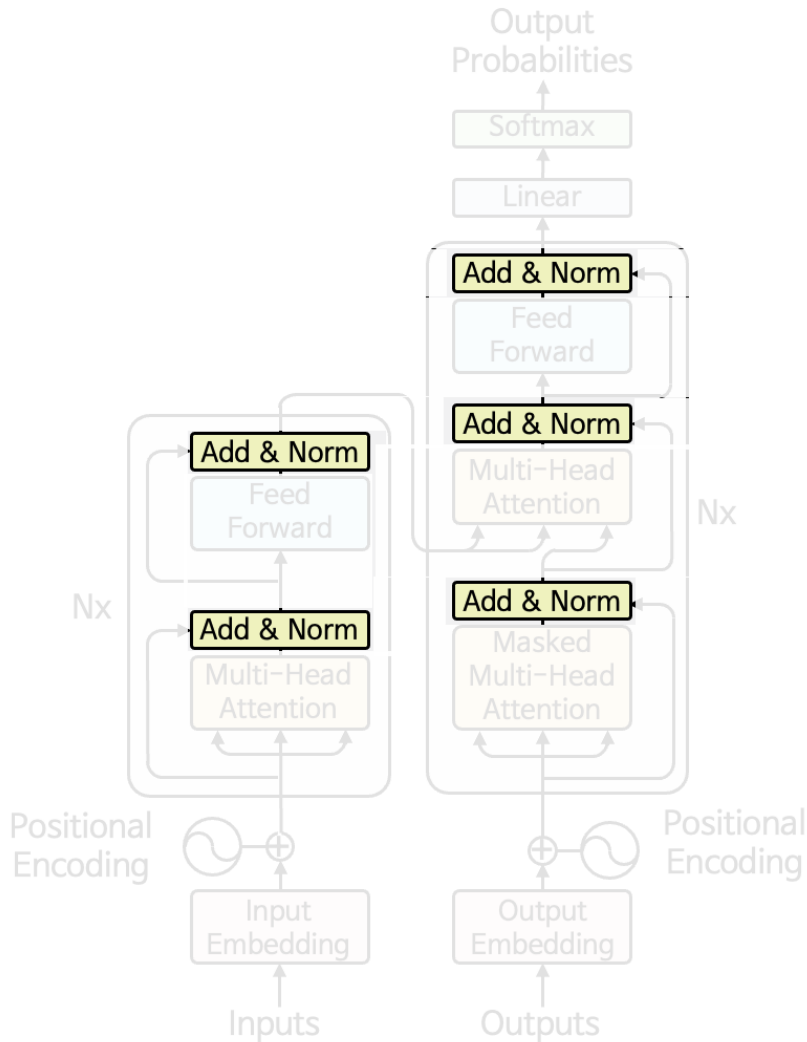
Embedding layer



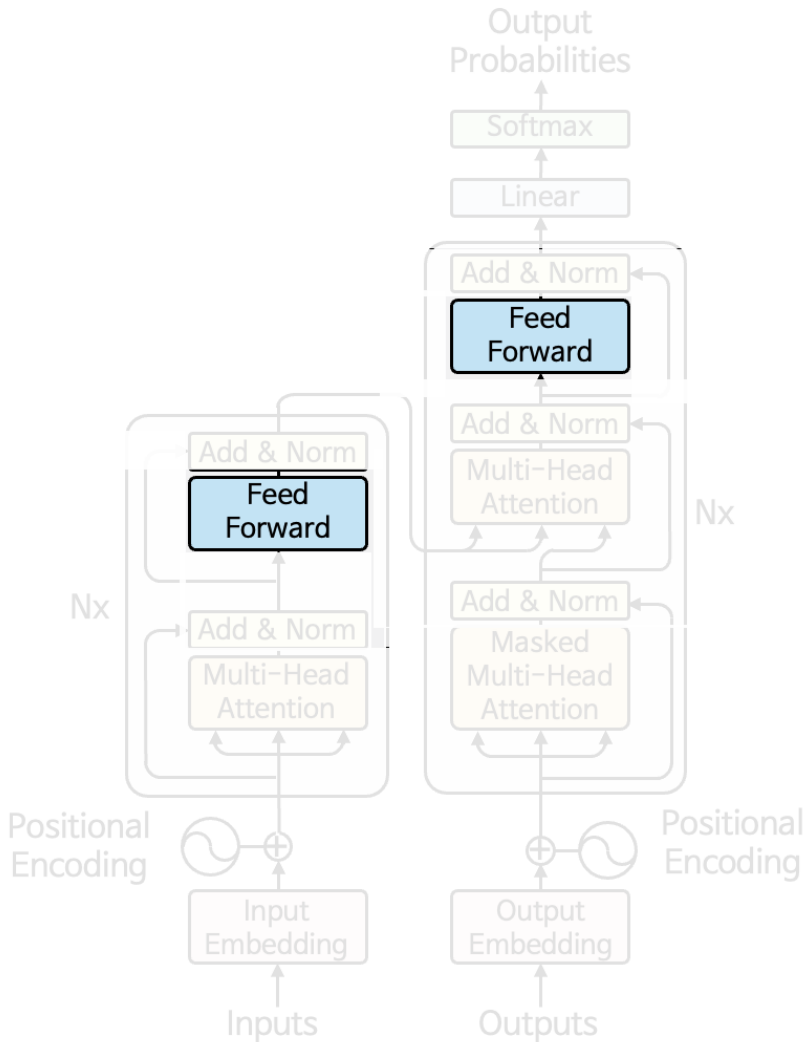
Multi-head attention



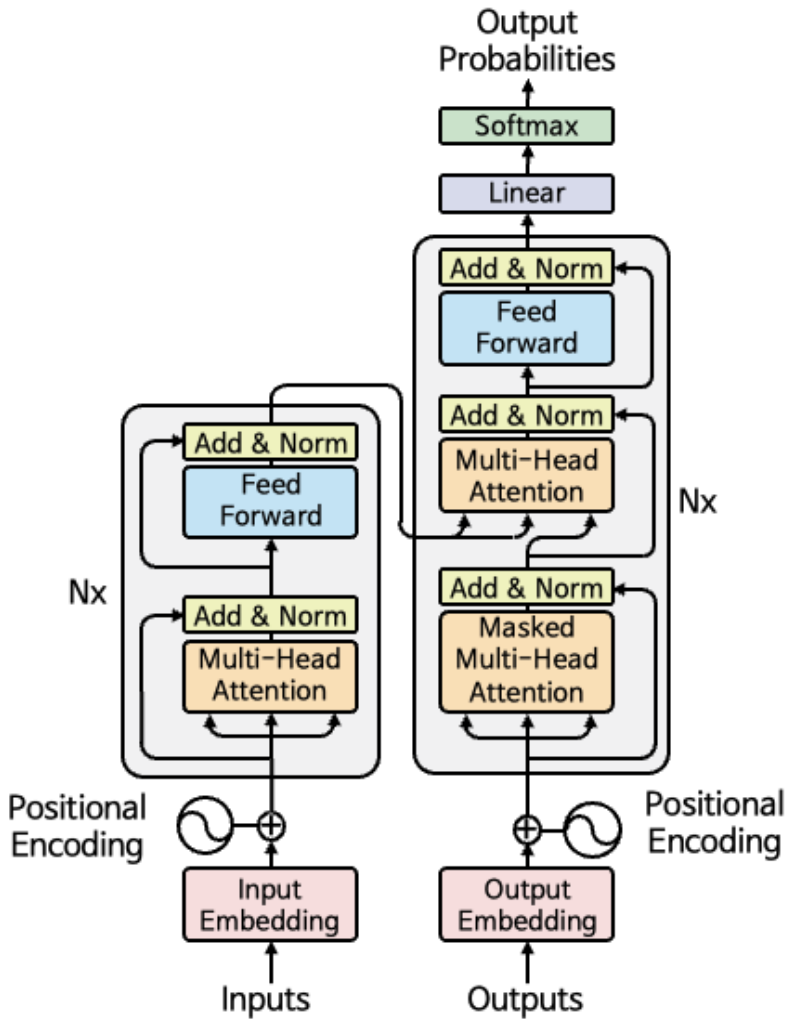
Add & Norm layer



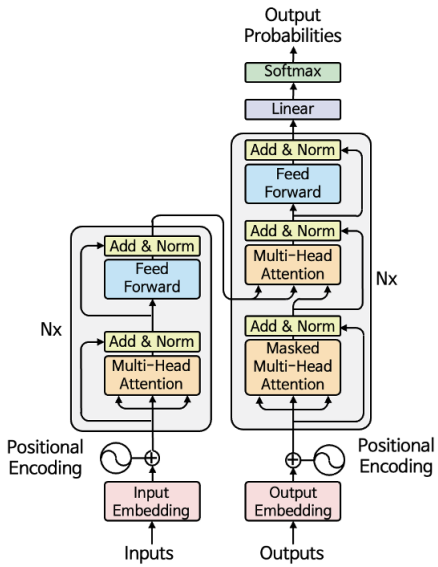
and also the Feed-Forward layer.



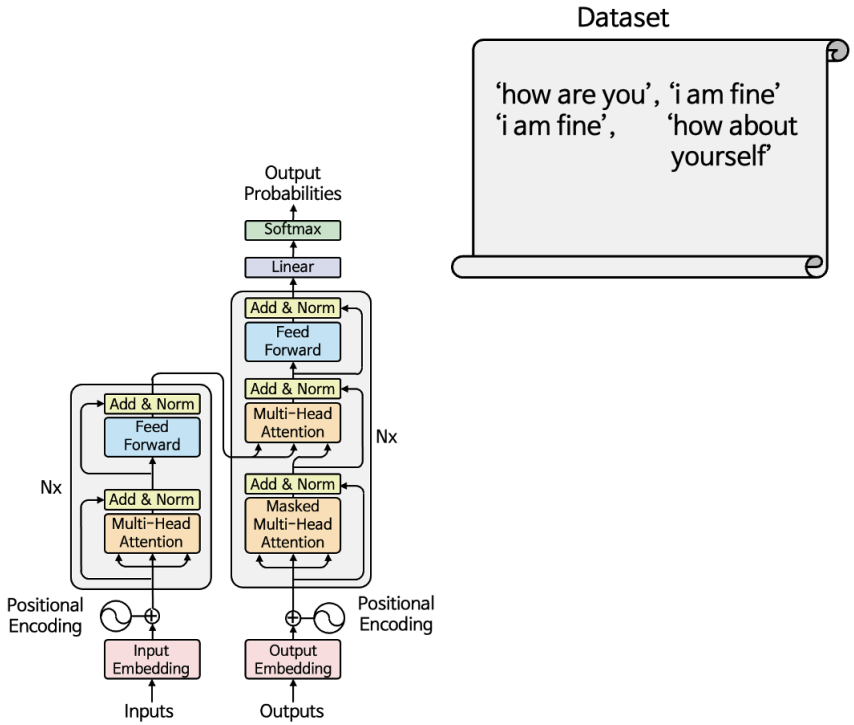
So, even though the Transformer may look complicated at first glance, it's actually made up of a few components that are repeatedly stacked.



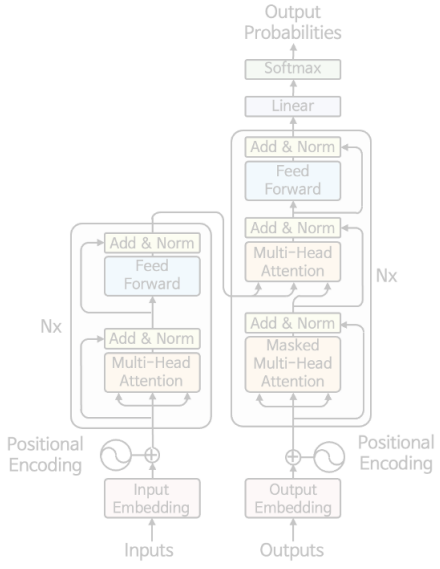
Let's use simple example to explore how the Transformer learns.



To train a Transformer model, (1) we create a dataset.



(2) We extract all words from the dataset to build a vocabulary list.



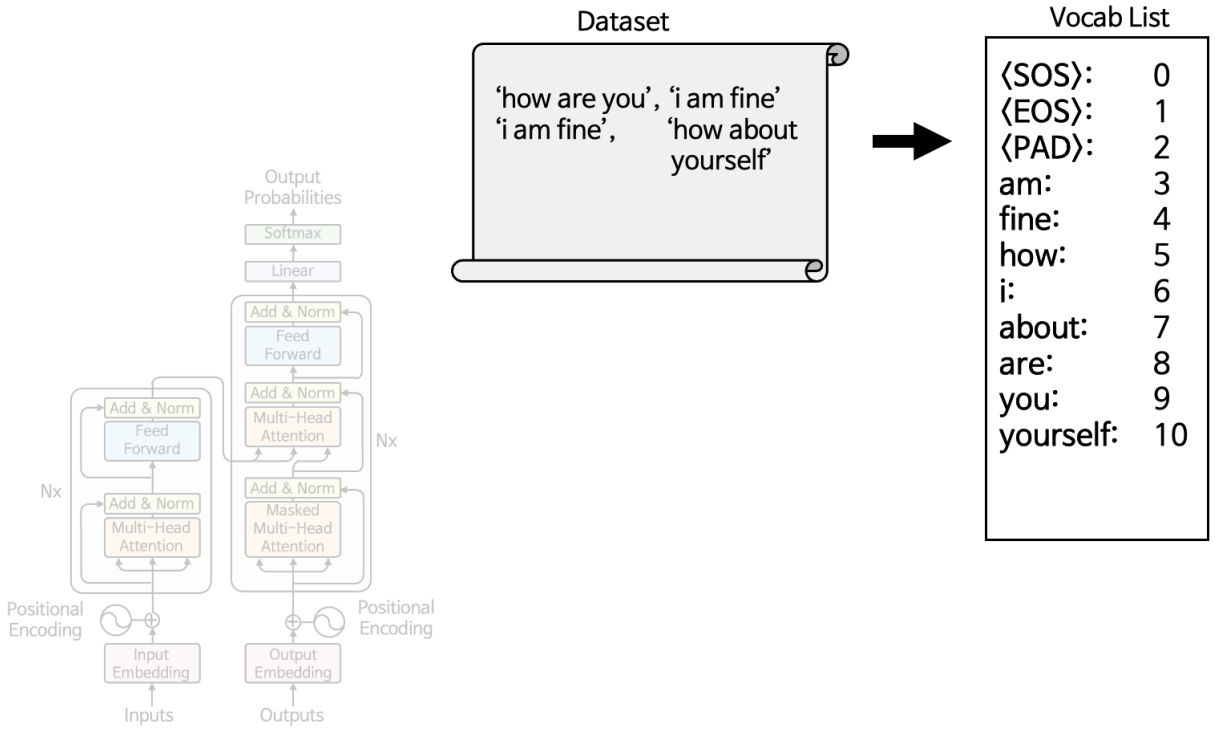
Dataset

'how are you', 'i am fine'
'i am fine', 'how about yourself'

Vocab List

⟨SOS⟩:
⟨EOS⟩:
⟨PAD⟩:
am:
fine:
how:
i:
about:
are:
you:
yourself:

(3) We assign each word (or token) a unique number so that the model can process it as input data.



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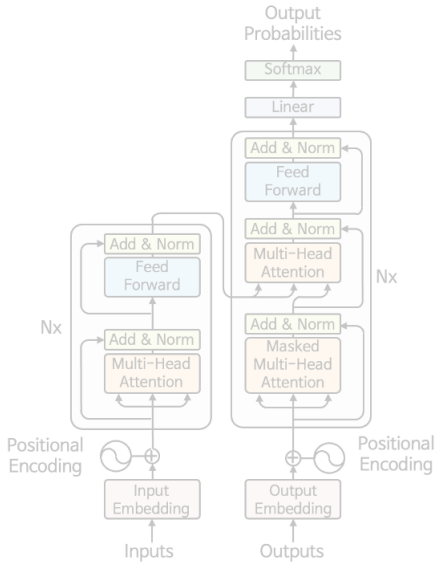


Dataset

'how are you', 'i am fine'
'i am fine', 'how about yourself'

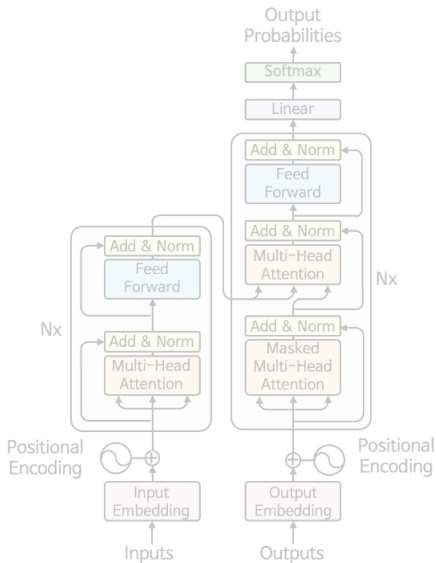
Vocab List

⟨SOS⟩:	0
⟨EOS⟩:	1
⟨PAD⟩:	2
am:	3
fine:	4
how:	5
i:	6
about:	7
are:	8
you:	9
yourself:	10

Suppose that our task is to make a model that can do such a Q&A

'How are you?' ➔ *'I am fine'*

We assume a very small number of tokens (for simplicity).



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'how are you', 'i am fine'
'i am fine', 'how about yourself'

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⟨SOS⟩:	0
⟨EOS⟩:	1
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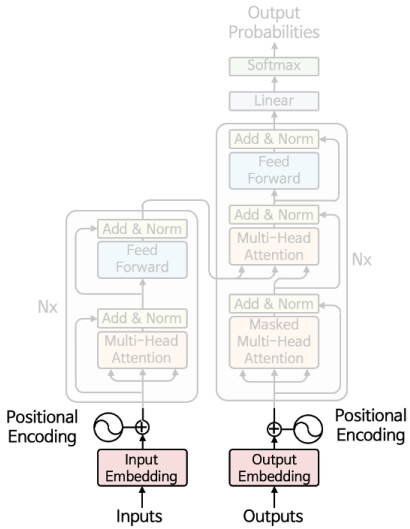
am:	3
fine:	4
how:	5
i:	6
about:	7
are:	8
you:	9
yourself:	10

Suppose that our task is to make a model that can do such a Q&A

'How are you?' ➔ *'<SOS> I am'*

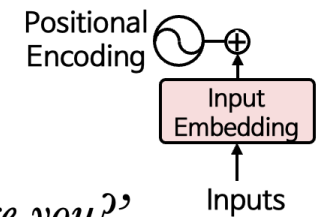
[5, 8, 9] ➔ [0, 6, 3]

(4) We transform input sentence → Word embedding

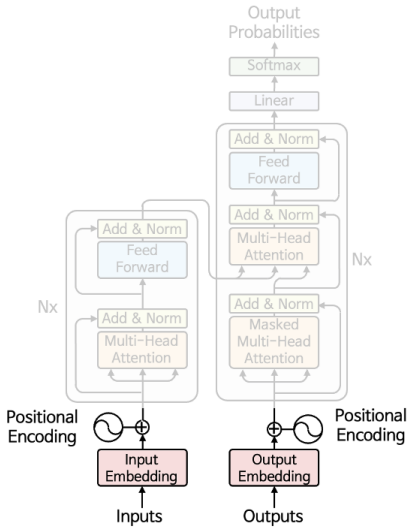


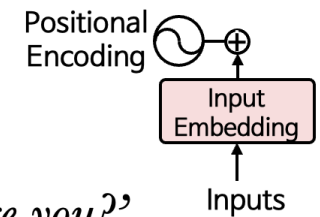
'How are you?'

[5, 8, 9]



The input embedding block takes input tokens like [5, 8, 9] and

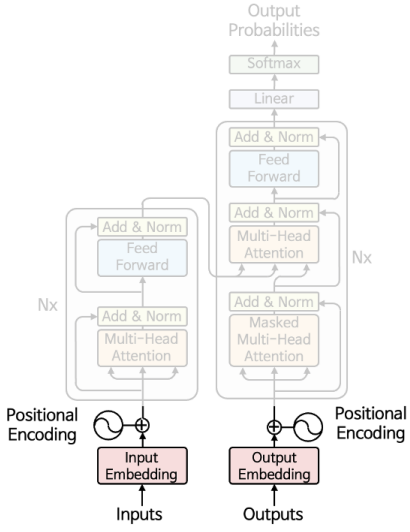




'How are you?'

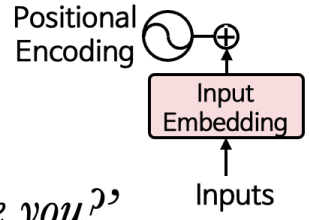
[5, 8, 9]

outputs the corresponding embedding vectors for each word.



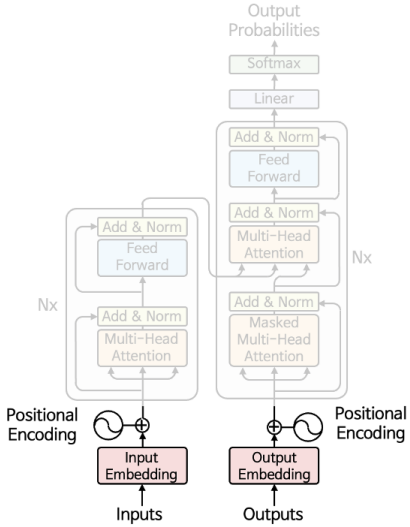
'How are you?'

[5, 8, 9]



5, how	-0.25	0.86	-1.38	-0.87	-0.22	1.72
8, are	-0.91	-0.66	2.22	0.52	0.35	-0.20
9, you	-1.05	1.28	0.15	0.23	0.06	0.43

The embedding layer compresses, for example, 11 words into

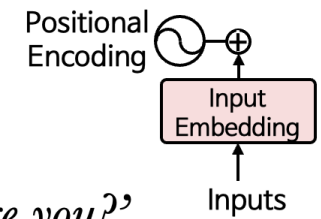


⟨SOS⟩:	0
⟨EOS⟩:	1
⟨PAD⟩:	2
am:	3
fine:	4
how:	5
i:	6
about:	7
are:	8
you:	9
yourself:	10

11

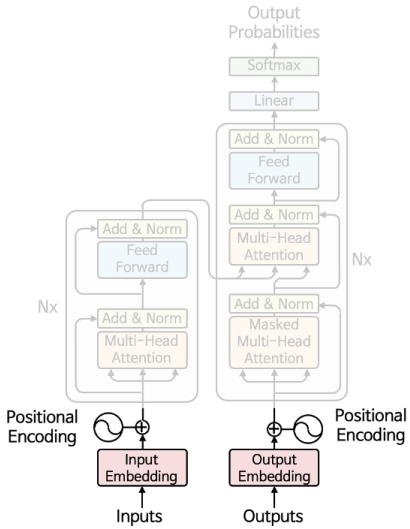
'How are you?'

[5, 8, 9]



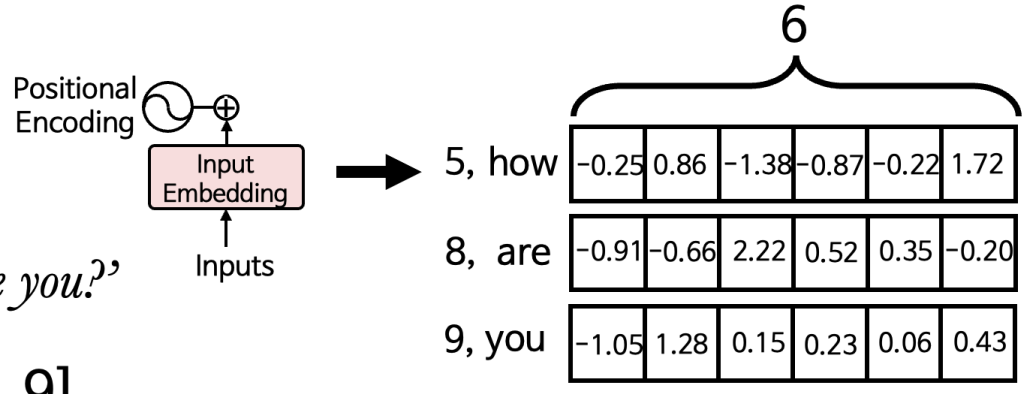
5, how	-0.25	0.86	-1.38	-0.87	-0.22	1.72
8, are	-0.91	-0.66	2.22	0.52	0.35	-0.20
9, you	-1.05	1.28	0.15	0.23	0.06	0.43

dense vectors of length 6 (*randomly initialized at the beginning of training; learned jointly with the rest of the model)

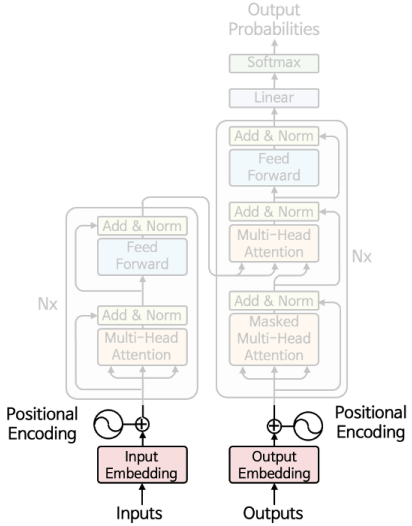


'How are you?'

[5, 8, 9]

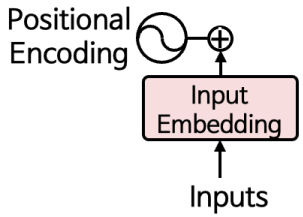


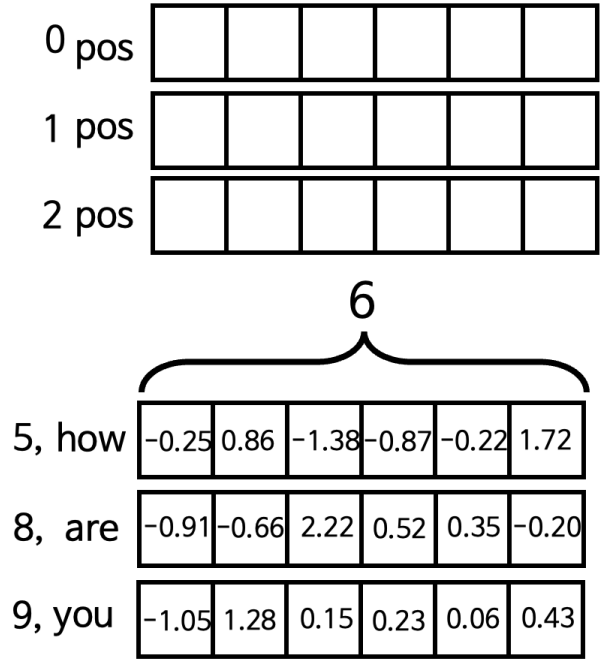
(5) We consider positional encoding.



⟨SOS⟩:	0
⟨EOS⟩:	1
⟨PAD⟩:	2
am:	3
fine:	4
how:	5
i:	6
about:	7
are:	8
you:	9
yourself:	10

11

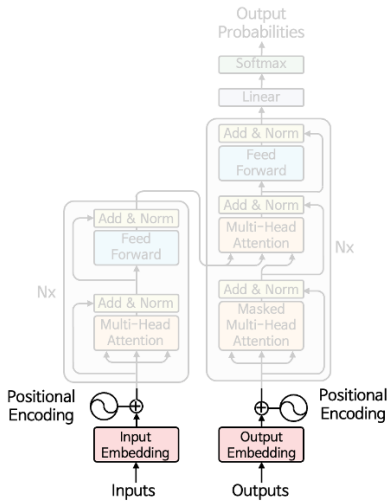


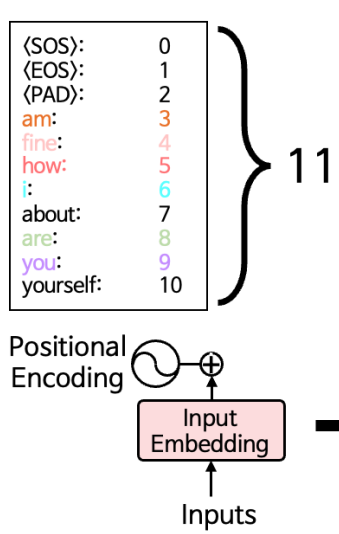


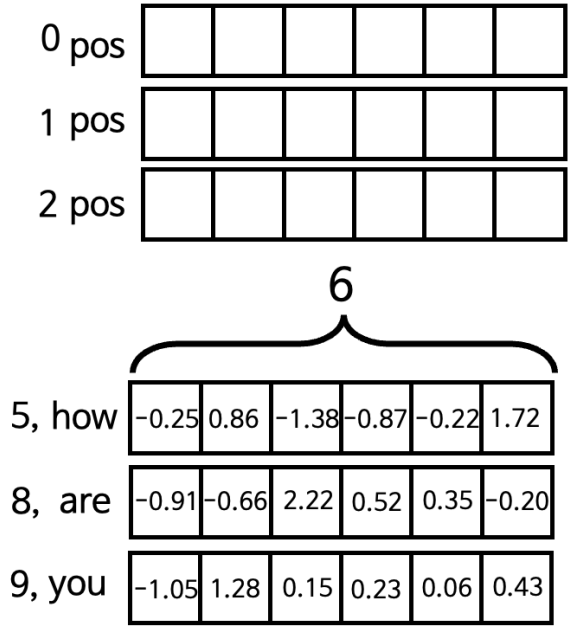
Why do we need this?

Why do we need this? Self-attention alone does not capture word order (e.g., RNNs); it treats inputs as a set, not a sequence.

$$PE_{(pos,2i)} = sin(\frac{pos}{10000^{\frac{2i}{d_{model}}}}) \quad PE_{(pos,2i+1)} = cos(\frac{pos}{10000^{\frac{2i}{d_{model}}}})$$







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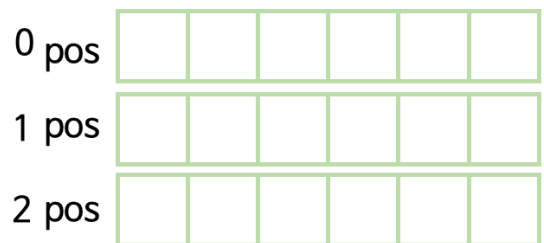
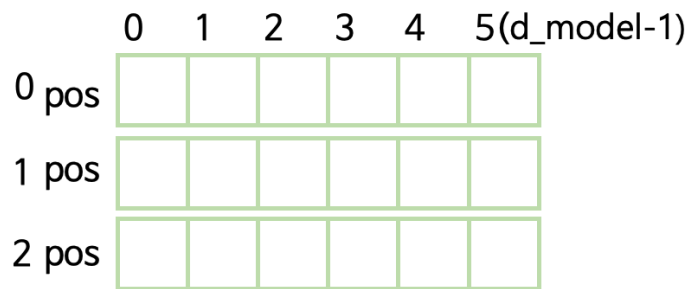
We encode the position of each word based on the following formulas.

$$PE_{(pos,2i)} = sin(\frac{pos}{10000^{\frac{2i}{d_{model}}}}) \qquad PE_{(pos,2i+1)} = cos(\frac{pos}{10000^{\frac{2i}{d_{model}}}})$$



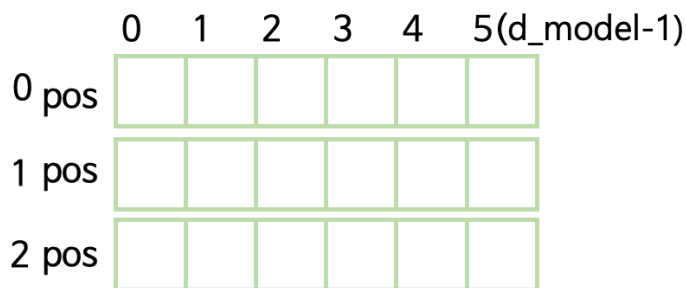
If you are interested in learning how the fomulas work, take a closer look.

$$PE_{(pos,2i)} = sin(\frac{pos}{10000^{\frac{2i}{d_{model}}}}) \quad PE_{(pos,2i+1)} = cos(\frac{pos}{10000^{\frac{2i}{d_{model}}}})$$

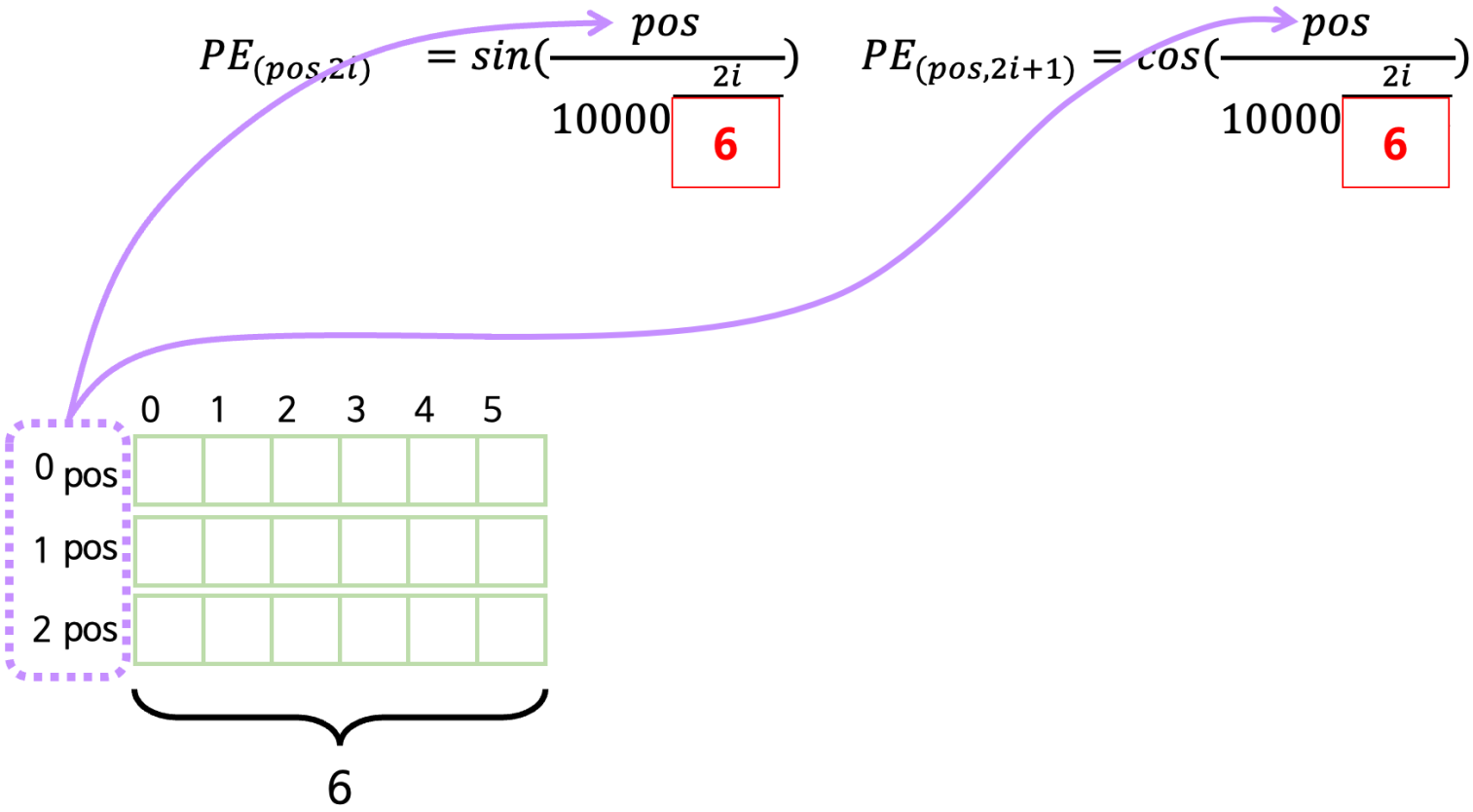


In this example, the value of d_{model} is 6 (the dimensionality of the model).

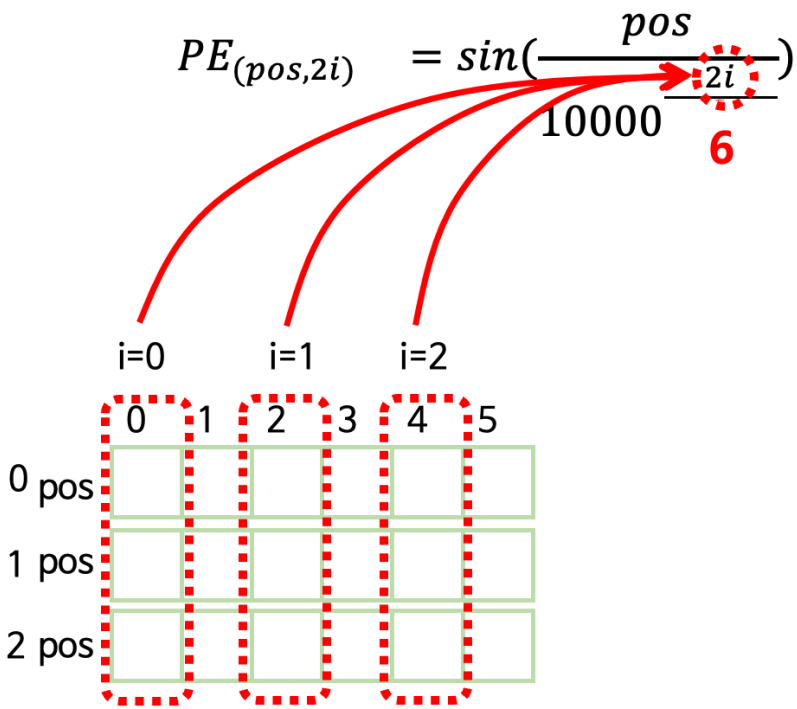
$$PE_{(pos,2i)} = sin(\frac{pos}{10000^{\frac{2i}{6}}}) \qquad PE_{(pos,2i+1)} = cos(\frac{pos}{10000^{\frac{2i}{6}}})$$



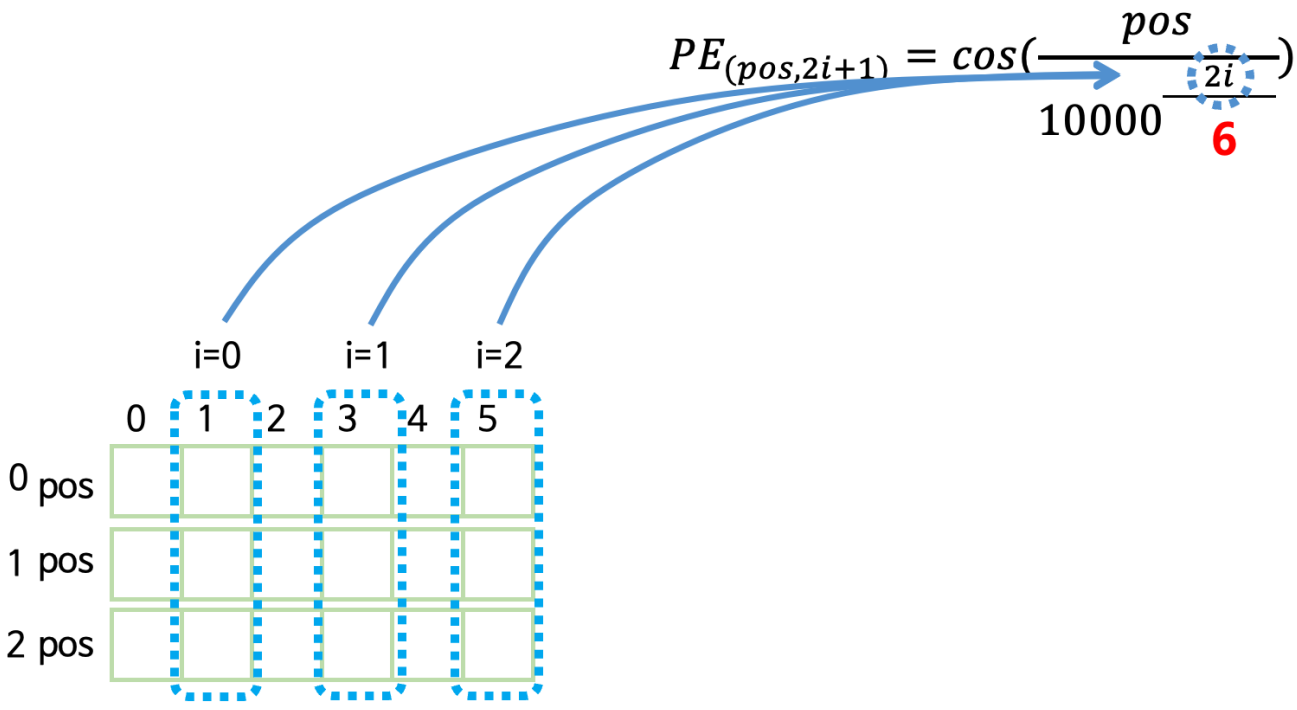
And the variable *pos* takes the values 0, 1, and 2 in order.



$i = 0, 1, 2$ denotes the dimension index used for even (2i) and odd (2i+1) components. For even dimensions, we apply this formula:



And for odd dimensions, we apply this formula:



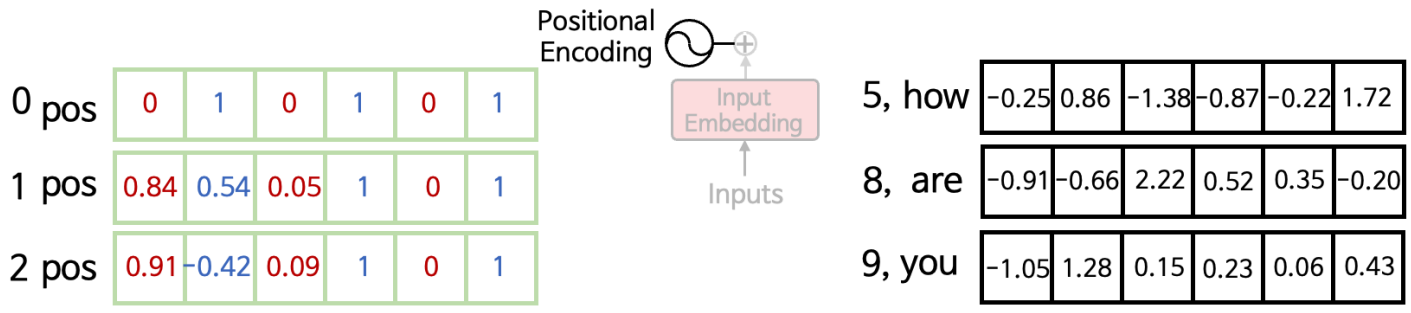
We can now compute the positional encoding values as follows:

$$PE_{(pos,2i)} = sin(\frac{pos}{10000^{\frac{2i}{d_{model}}}}) \qquad PE_{(pos,2i+1)} = cos(\frac{pos}{10000^{\frac{2i}{d_{model}}}})$$

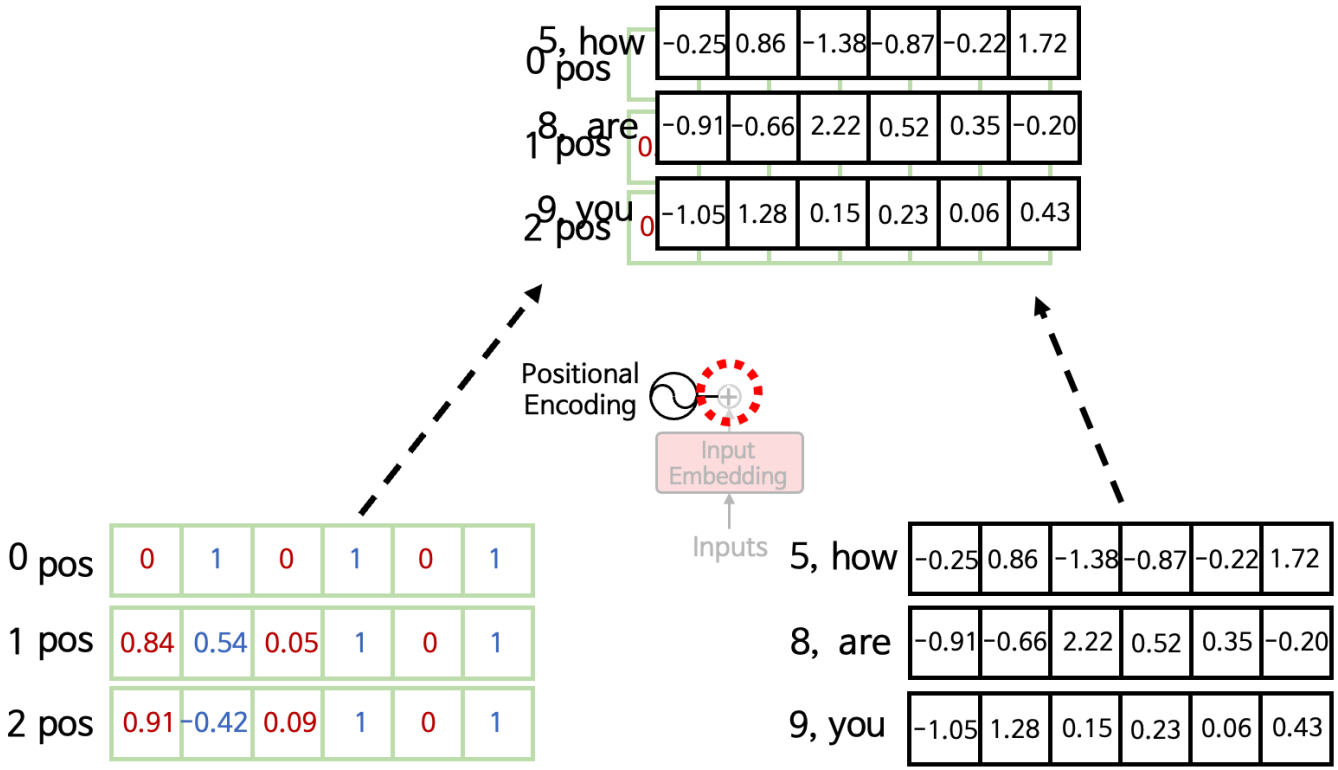
	0	1	2	3	4	5
0 pos	0	1	0	1	0	1
1 pos	0.84	0.54	0.05	1	0	1
2 pos	0.91	-0.42	0.09	1	0	1

6

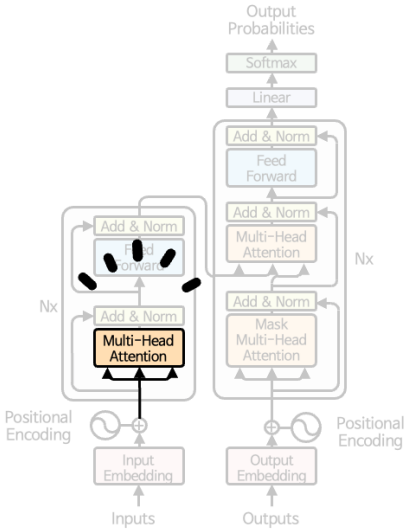
(6) Once we have these positional embeddings, we simply add them to the input embeddings:



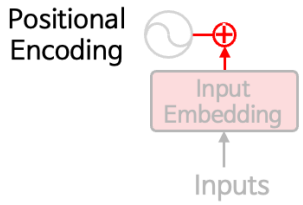
By adding them together, we obtain the combined input + positional embedding vectors.



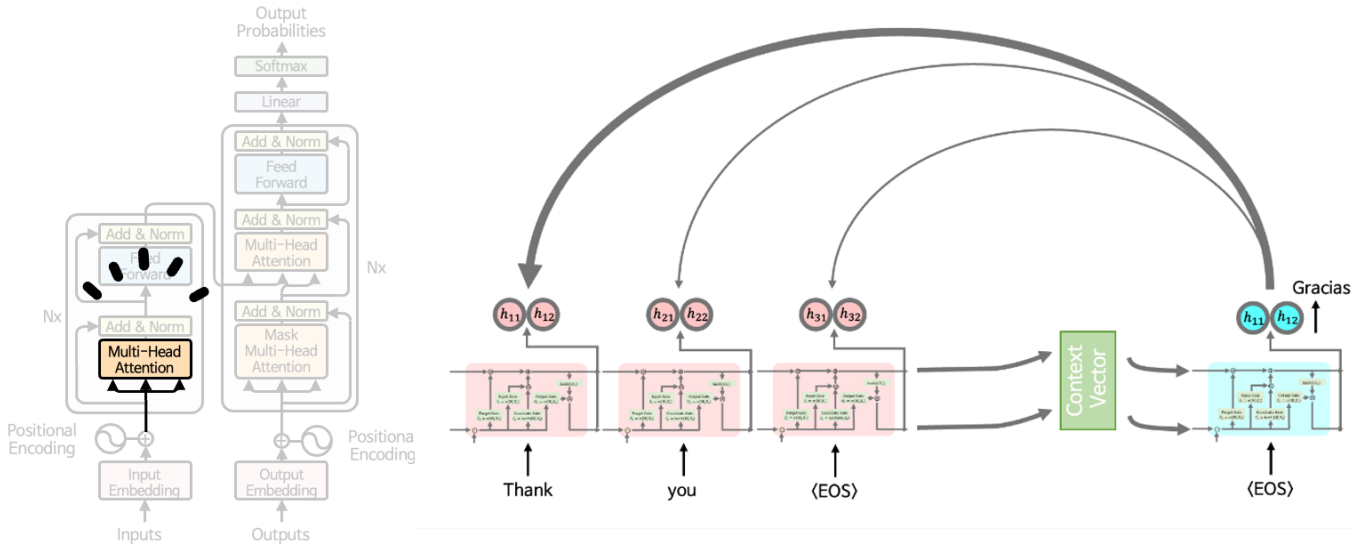
(7) We feed this combined input–position matrix into the multi-head attention.



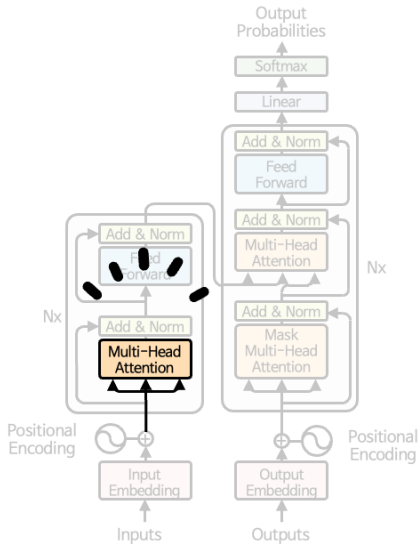
-0.25	1.86	-1.38	0.13	-0.22	2.72
-0.07	-0.12	2.26	1.52	0.35	0.80
-0.15	0.86	0.24	1.23	0.06	1.43

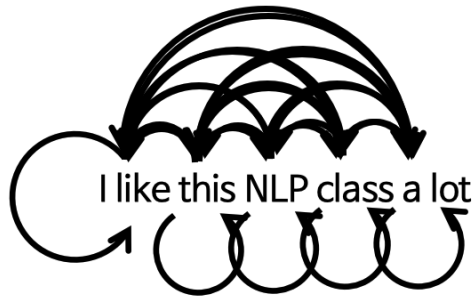


Transformer's multi-head attention is different from the attention mechanism used in traditional seq2seq models. While seq2seq attention focuses on aligning the input and output sequences,

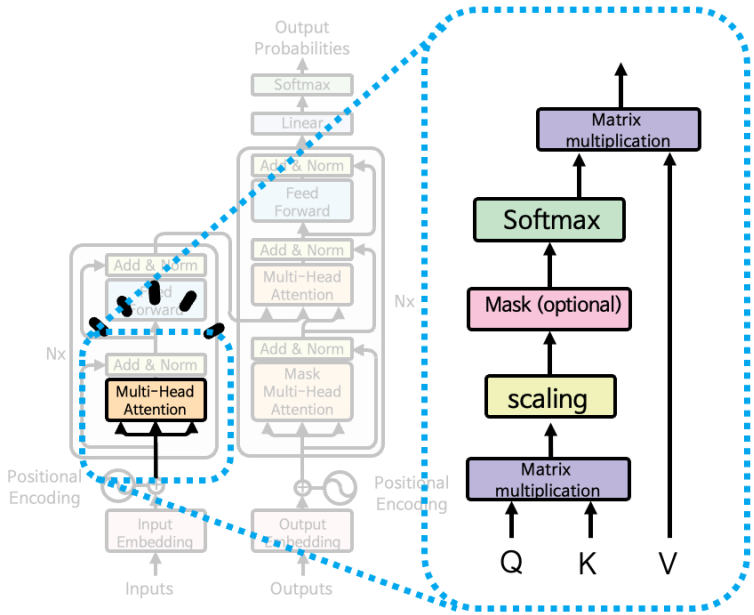


the Transformer's attention captures the relationships between words within the same input sentence.

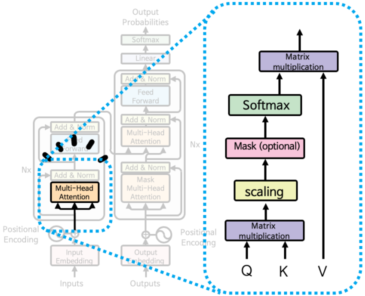




The structure of the multi-head attention mechanism used for self-attention looks like this:



We make three copies of the input + positional encoding matrix.

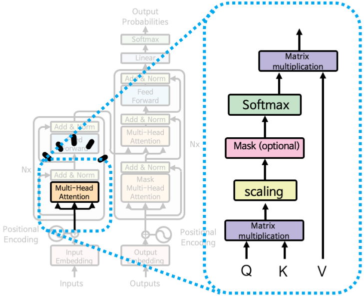


-0.25	1.86	-1.38	0.13	-0.22	2.72
-0.07	-0.12	2.26	1.52	0.35	0.80
-0.15	0.86	0.24	1.23	0.06	1.43

-0.25	1.86	-1.38	0.13	-0.22	2.72
-0.07	-0.12	2.26	1.52	0.35	0.80
-0.15	0.86	0.24	1.23	0.06	1.43

-0.25	1.86	-1.38	0.13	-0.22	2.72
-0.07	-0.12	2.26	1.52	0.35	0.80
-0.15	0.86	0.24	1.23	0.06	1.43

We make three copies of the input + positional encoding matrix.



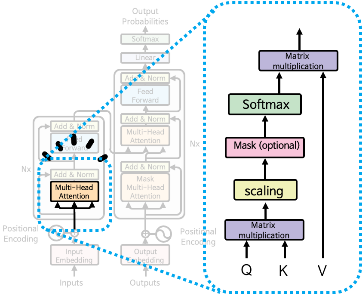
-0.25	1.86	-1.38	0.13	-0.22	2.72
-0.07	-0.12	2.26	1.52	0.35	0.80
-0.15	0.86	0.24	1.23	0.06	1.43

-0.25	1.86	-1.38	0.13	-0.22	2.72
-0.07	-0.12	2.26	1.52	0.35	0.80
-0.15	0.86	0.24	1.23	0.06	1.43

-0.25	1.86	-1.38	0.13	-0.22	2.72
-0.07	-0.12	2.26	1.52	0.35	0.80
-0.15	0.86	0.24	1.23	0.06	1.43

Why?

We make three copies of the input + positional encoding matrix.



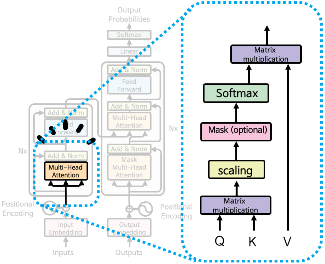
-0.25	1.86	-1.38	0.13	-0.22	2.72
-0.07	-0.12	2.26	1.52	0.35	0.80
-0.15	0.86	0.24	1.23	0.06	1.43

-0.25	1.86	-1.38	0.13	-0.22	2.72
-0.07	-0.12	2.26	1.52	0.35	0.80
-0.15	0.86	0.24	1.23	0.06	1.43

-0.25	1.86	-1.38	0.13	-0.22	2.72
-0.07	-0.12	2.26	1.52	0.35	0.80
-0.15	0.86	0.24	1.23	0.06	1.43

Why? This is done to create Query (Q), Key (K), and Value (V) matrices — each representing a different projection of the same input for the attention mechanism. Q and K determine how much attention each token should pay to others, while V carries the actual information that is aggregated.

To obtain the Q matrix, we create the following 6×6 weight matrix (randomly initialized).



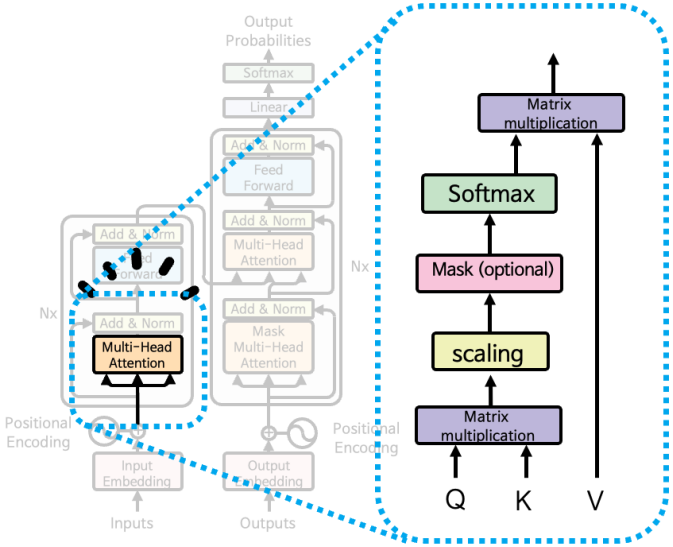
−0.25	1.86	−1.38	0.13	−0.22	2.72
−0.07	−0.12	2.26	1.52	0.35	0.80
−0.15	0.86	0.24	1.23	0.06	1.43

0.37	0.37	−0.19	0.23	0.32	0.21
−0.30	−0.09	0.35	−0.10	−0.38	0.21
0.22	0.05	0.09	−0.23	0.09	0.15
0.14	−0.36	−0.23	−0.14	0.21	−0.07
0.13	0.17	−0.21	−0.30	0.07	−0.11
−0.22	−0.06	−0.02	0.15	−0.15	0.04

−0.25	1.86	−1.38	0.13	−0.22	2.72
−0.07	−0.12	2.26	1.52	0.35	0.80
−0.15	0.86	0.24	1.23	0.06	1.43

−0.25	1.86	−1.38	0.13	−0.22	2.72
−0.07	−0.12	2.26	1.52	0.35	0.80
−0.15	0.86	0.24	1.23	0.06	1.43

And then we perform matrix multiplication to obtain the Q (Query) matrix.



-0.25	1.86	-1.38	0.13	-0.22	2.72
-0.07	-0.12	2.26	1.52	0.35	0.80
-0.15	0.86	0.24	1.23	0.06	1.43

X

0.37	0.37	-0.19	0.23	0.32	0.21
-0.30	-0.09	0.35	-0.10	-0.38	0.21
0.22	0.05	0.09	-0.23	0.09	0.15
0.14	-0.36	-0.23	-0.14	0.21	-0.07
0.13	0.17	-0.21	-0.30	0.07	-0.11
-0.22	-0.06	-0.02	0.15	-0.15	0.04

=

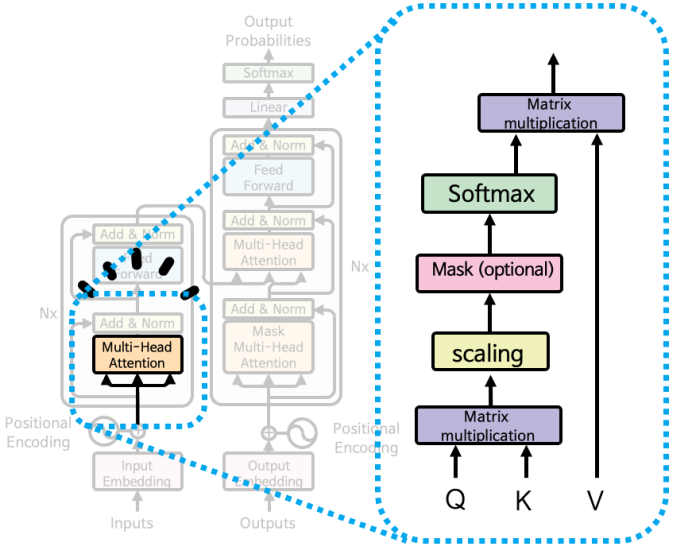
Q

-1.56	-0.25	0.54	0.52	-1.31	0.27
0.59	-0.46	-0.25	-0.73	0.47	0.19
-0.39	-0.48	0.03	-0.16	-0.30	0.16

-0.25	1.86	-1.38	0.13	-0.22	2.72
-0.07	-0.12	2.26	1.52	0.35	0.80
-0.15	0.86	0.24	1.23	0.06	1.43

-0.25	1.86	-1.38	0.13	-0.22	2.72
-0.07	-0.12	2.26	1.52	0.35	0.80
-0.15	0.86	0.24	1.23	0.06	1.43

To compute the K (Key) matrix, we create another 6×6 weight matrix (randomly initialized) and multiply it with the input.



-0.25	1.86	-1.38	0.13	-0.22	2.72
-0.07	-0.12	2.26	1.52	0.35	0.80
-0.15	0.86	0.24	1.23	0.06	1.43

-0.25	1.86	-1.38	0.13	-0.22	2.72
-0.07	-0.12	2.26	1.52	0.35	0.80
-0.15	0.86	0.24	1.23	0.06	1.43

X

-0.18	-0.28	0.35	-0.34	0.30	0.37
-0.24	-0.24	0.13	-0.41	-0.30	-0.32
0.00	-0.14	-0.35	0.12	-0.07	0.10
-0.15	-0.32	0.28	-0.09	0.09	-0.18
-0.03	0.34	-0.11	0.16	0.21	-0.04
-0.28	-0.08	-0.16	-0.34	0.33	-0.31

=

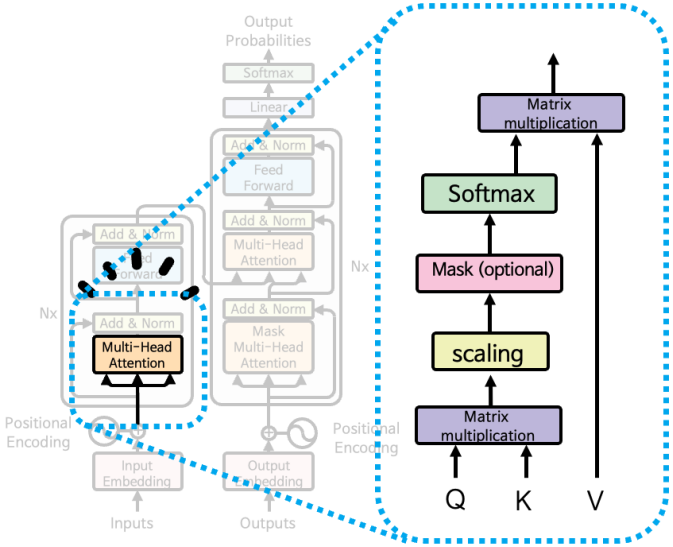
Q

-1.56	-0.25	0.54	0.52	-1.31	0.27
0.59	-0.46	-0.25	-0.73	0.47	0.19
-0.39	-0.48	0.03	-0.16	-0.30	0.16

K

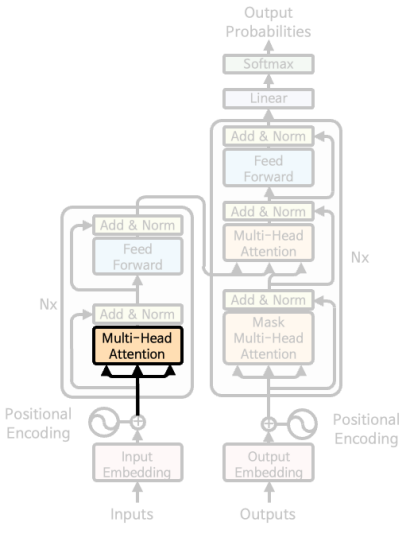
-1.18	-0.52	0.26	-1.79	0.32	-1.68
-0.42	-0.70	-0.56	-0.01	0.32	-0.28
-0.77	-0.69	0.09	-0.85	0.27	-0.96

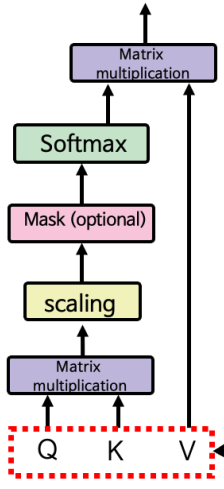
Using the same process, we perform another matrix multiplication to obtain the V (Value) matrix.



$$
\begin{bmatrix} -0.25 & 1.86 & -1.38 & 0.13 & -0.22 & 2.72 \\ -0.07 & -0.12 & 2.26 & 1.52 & 0.35 & 0.80 \\ -0.15 & 0.86 & 0.24 & 1.23 & 0.06 & 1.43 \end{bmatrix}
\times
\begin{bmatrix} 0.22 & -0.21 & -0.30 & -0.23 & -0.35 & 0.13 \\ -0.17 & 0.12 & -0.22 & 0.10 & -0.22 & -0.38 \\ 0.25 & 0.09 & 0.37 & -0.05 & -0.36 & -0.27 \\ -0.10 & -0.10 & -0.14 & -0.30 & -0.26 & -0.14 \\ 0.23 & 0.24 & -0.14 & 0.01 & 0.41 & 0.06 \\ -0.32 & 0.28 & -0.40 & -0.28 & 0.08 & -0.36 \end{bmatrix}
=
\begin{matrix} Q \\ \begin{bmatrix} -1.56 & -0.25 & 0.54 & 0.52 & -1.31 & 0.27 \\ 0.59 & -0.46 & -0.25 & -0.73 & 0.47 & 0.19 \\ -0.39 & -0.48 & 0.03 & -0.16 & -0.30 & 0.16 \end{bmatrix} \\ K \\ \begin{bmatrix} -1.18 & -0.52 & 0.26 & -1.79 & 0.32 & -1.68 \\ -0.42 & -0.70 & -0.56 & -0.01 & 0.32 & -0.28 \\ -0.77 & -0.69 & 0.09 & -0.85 & 0.27 & -0.96 \end{bmatrix} \\ V \\ \begin{bmatrix} -1.63 & 0.85 & -1.91 & -0.47 & 0.25 & -1.38 \\ 0.24 & 0.34 & 0.32 & -0.79 & -0.95 & -1.04 \\ -0.67 & 0.44 & -0.80 & -0.65 & -0.41 & -1.09 \end{bmatrix} \end{matrix}
$$

Now, we have the three inputs of the multi-head attention layer — Q (Query), K (Key), and V (Value).





Q

-1.56	-0.25	0.54	0.52	-1.31	0.27
0.59	-0.46	-0.25	-0.73	0.47	0.19
-0.39	-0.48	0.03	-0.16	-0.30	0.16

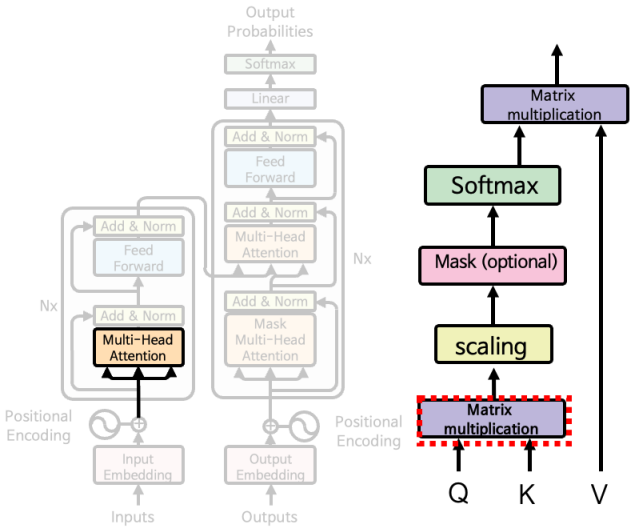
K

-1.18	-0.52	0.26	-1.79	0.32	-1.68
-0.42	-0.70	-0.56	-0.01	0.32	-0.28
-0.77	-0.69	0.09	-0.85	0.27	-0.96

V

-1.63	0.85	-1.91	-0.47	0.25	-1.38
0.24	0.34	0.32	-0.79	-0.95	-1.04
-0.67	0.44	-0.80	-0.65	-0.41	-1.09

Next, we perform the matrix multiplication between Q and K. The formula for this operation is as follows:



$$Q \cdot K^T$$

Q

-1.56	-0.25	0.54	0.52	-1.31	0.27
0.59	-0.46	-0.25	-0.73	0.47	0.19
-0.39	-0.48	0.03	-0.16	-0.30	0.16

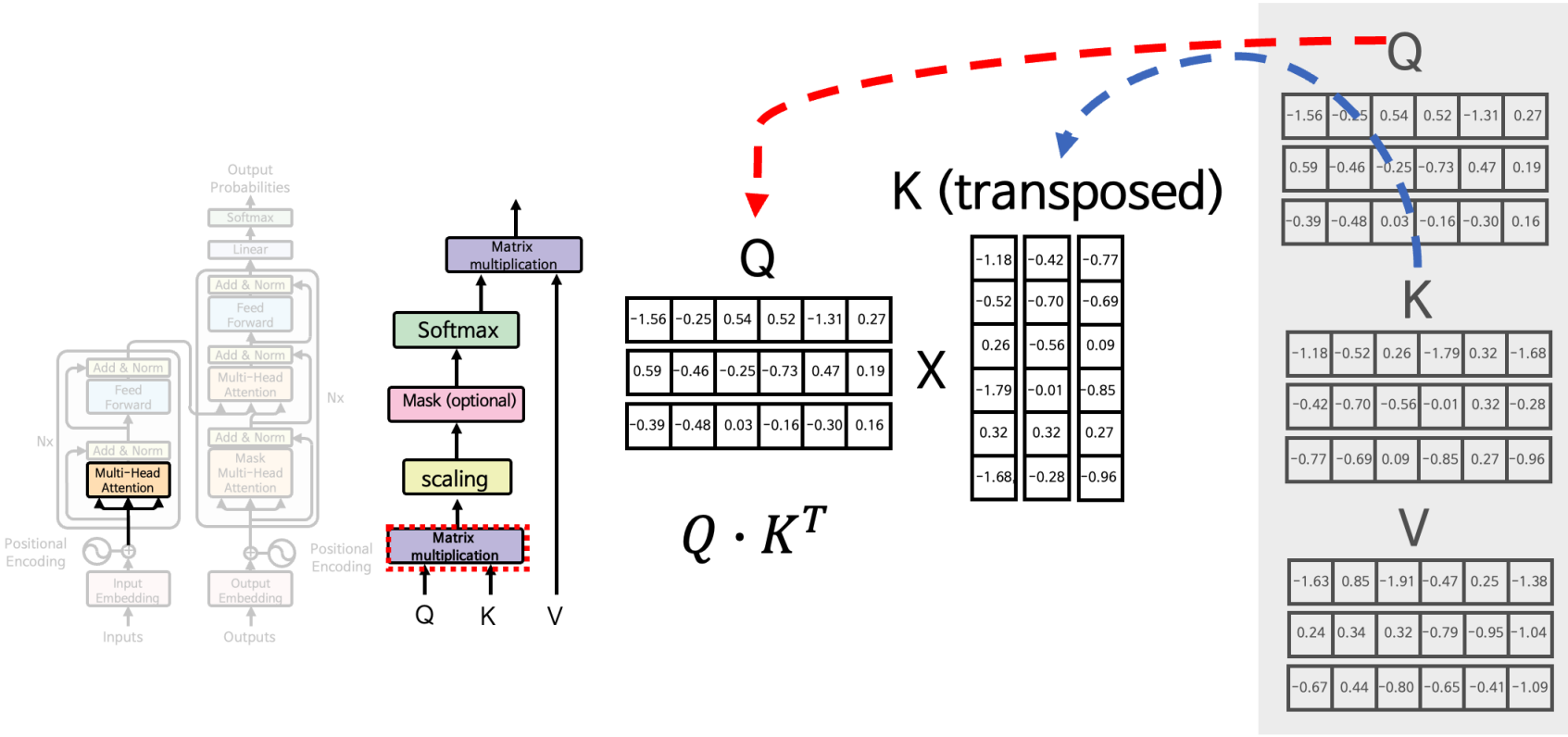
K

-1.18	-0.52	0.26	-1.79	0.32	-1.68
-0.42	-0.70	-0.56	-0.01	0.32	-0.28
-0.77	-0.69	0.09	-0.85	0.27	-0.96

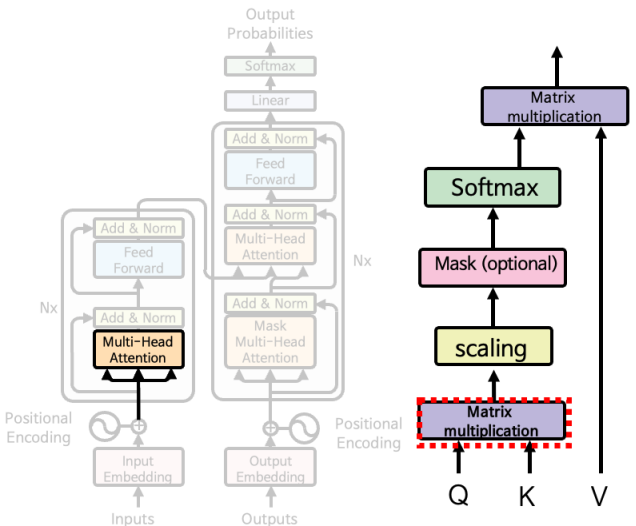
V

-1.63	0.85	-1.91	-0.47	0.25	-1.38
0.24	0.34	0.32	-0.79	-0.95	-1.04
-0.67	0.44	-0.80	-0.65	-0.41	-1.09

When we plug in the matrix values and calculate,



we can obtain the result as follows:



$$Q \cdot K^T$$

Q

-1.56	-0.25	0.54	0.52	-1.31	0.27
0.59	-0.46	-0.25	-0.73	0.47	0.19
-0.39	-0.48	0.03	-0.16	-0.30	0.16

X

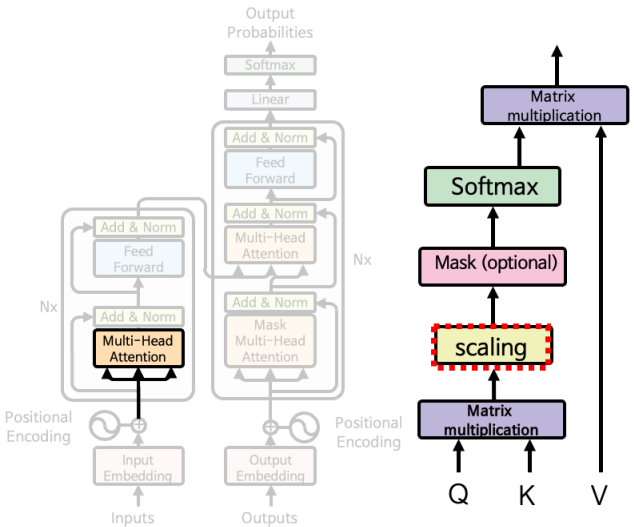
K (transposed)

-1.18	-0.42	-0.77
-0.52	-0.70	-0.69
0.26	-0.56	0.09
-1.79	-0.01	-0.85
0.32	0.32	0.27
-1.68	-0.28	-0.96

=

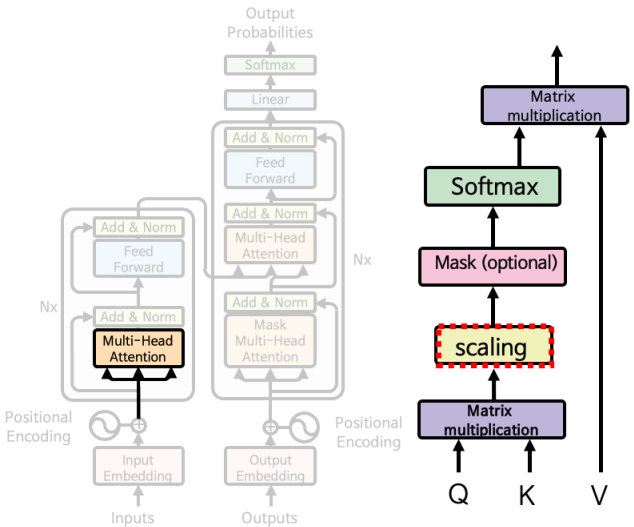
0.31	0.03	0.37
0.61	0.32	0.40
0.64	0.35	0.54

Next, we apply *scaling*, which divides the matrix by $\sqrt{6}$, since in this example the value of d_{model} is 6.



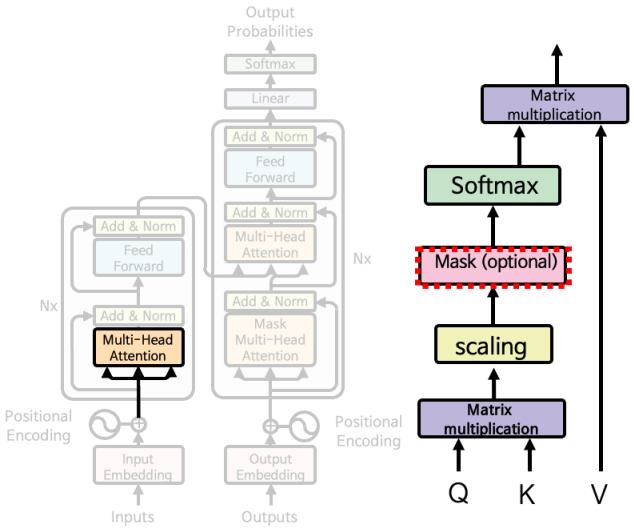
$$Q \cdot K^T = \begin{array}{|c|c|c|}\hline 0.31 & 0.03 & 0.37 \\ \hline 0.61 & 0.32 & 0.40 \\ \hline 0.64 & 0.35 & 0.54 \\ \hline \end{array} \div \sqrt{6}$$

Next, we apply *scaling*, which divides the matrix by $\sqrt{6}$, since in this example the value of d_{model} is 6.



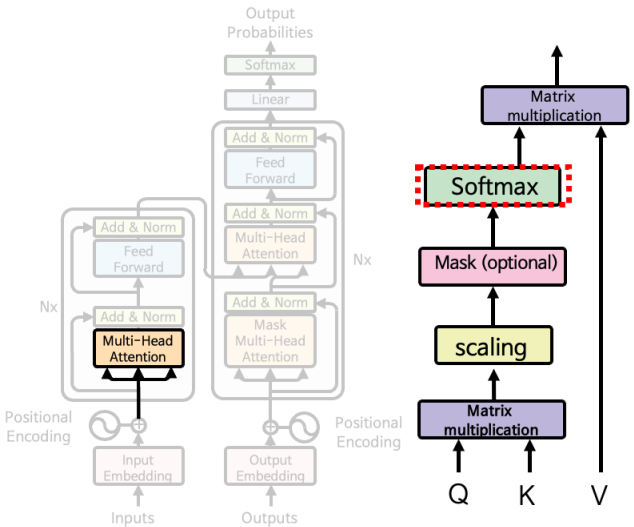
$$Q \cdot K^T = \begin{bmatrix} 0.31 & 0.03 & 0.37 \\ 0.61 & 0.32 & 0.40 \\ 0.64 & 0.35 & 0.54 \end{bmatrix} \div \sqrt{6} = \begin{bmatrix} 0.13 & 0.01 & 0.15 \\ 0.25 & 0.13 & 0.16 \\ 0.26 & 0.14 & 0.22 \end{bmatrix}$$

Since the mask layer is not used in the encoder, we will skip it here.



$$Q \cdot K^T = \begin{bmatrix} 0.31 & 0.03 & 0.37 \\ 0.61 & 0.32 & 0.40 \\ 0.64 & 0.35 & 0.54 \end{bmatrix} \div \sqrt{6} = \begin{bmatrix} 0.13 & 0.01 & 0.15 \\ 0.25 & 0.13 & 0.16 \\ 0.26 & 0.14 & 0.22 \end{bmatrix}$$

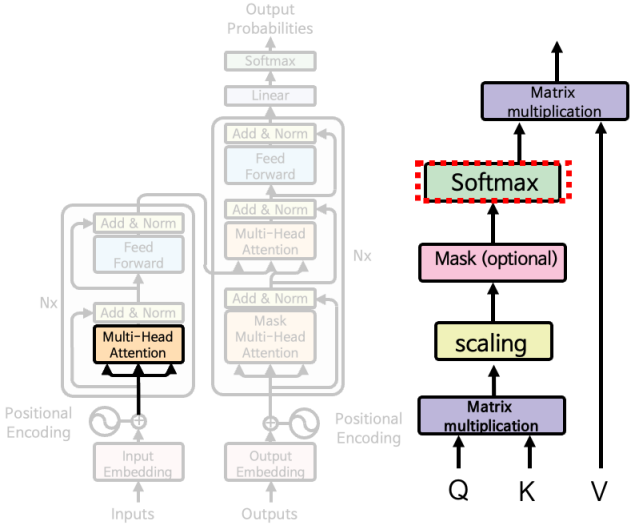
The next softmax layer converts the matrix values into probabilities.



$$softmax\left(\begin{array}{|c|c|c|}\hline 0.13 & 0.01 & 0.15 \\ \hline 0.25 & 0.13 & 0.16 \\ \hline 0.26 & 0.14 & 0.22 \\ \hline \end{array}\right) = \begin{array}{|c|c|c|}\hline 0.34 & 0.31 & 0.35 \\ \hline 0.36 & 0.32 & 0.33 \\ \hline 0.35 & 0.31 & 0.34 \\ \hline \end{array}$$

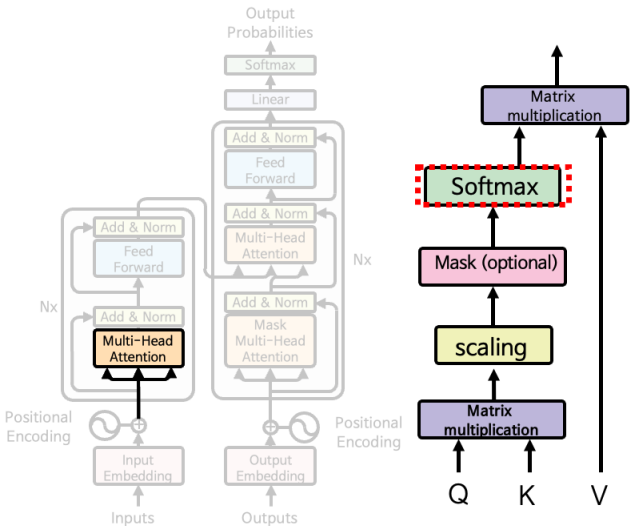
$$Q \cdot K^T = \begin{array}{|c|c|c|}\hline 0.31 & 0.03 & 0.37 \\ \hline 0.61 & 0.32 & 0.40 \\ \hline 0.64 & 0.35 & 0.54 \\ \hline \end{array} \div \sqrt{6} = \begin{array}{|c|c|c|}\hline 0.13 & 0.01 & 0.15 \\ \hline 0.25 & 0.13 & 0.16 \\ \hline 0.26 & 0.14 & 0.22 \\ \hline \end{array}$$

This 3×3 matrix represents the self-attention weights.



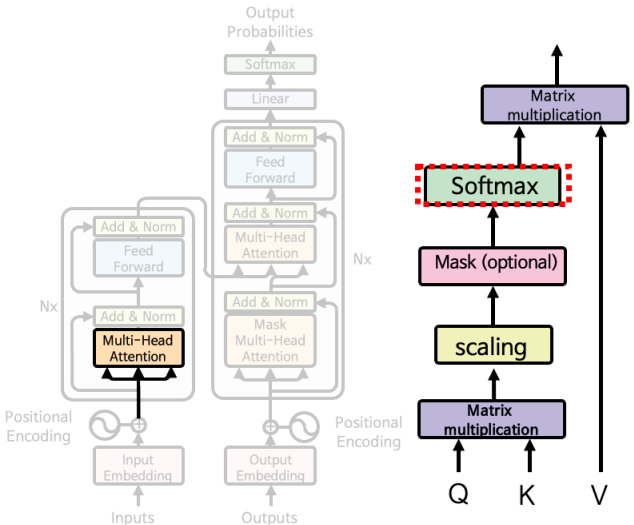
$$softmax\left(\begin{array}{|c|c|c|}\hline 0.13 & 0.01 & 0.15 \\ \hline 0.25 & 0.13 & 0.16 \\ \hline 0.26 & 0.14 & 0.22 \\ \hline\end{array}\right) = \begin{array}{c|c|c|c|}\multicolumn{1}{c}{} & \text{how} & \text{are} & \text{you} \\ \cline{2-4} \text{how} & 0.34 & 0.31 & 0.35 \\ \cline{2-4} \text{are} & 0.36 & 0.32 & 0.33 \\ \cline{2-4} \text{you} & 0.35 & 0.31 & 0.34 \\ \cline{2-4}\end{array}$$

This matrix numerically shows how each word in the input is related to every other word.



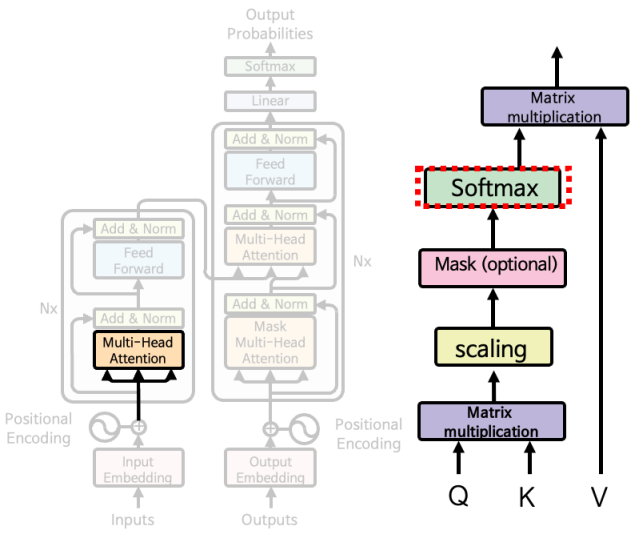
	how	are	you
how	0.34	0.31	0.35
are	0.36	0.32	0.33
you	0.35	0.31	0.34

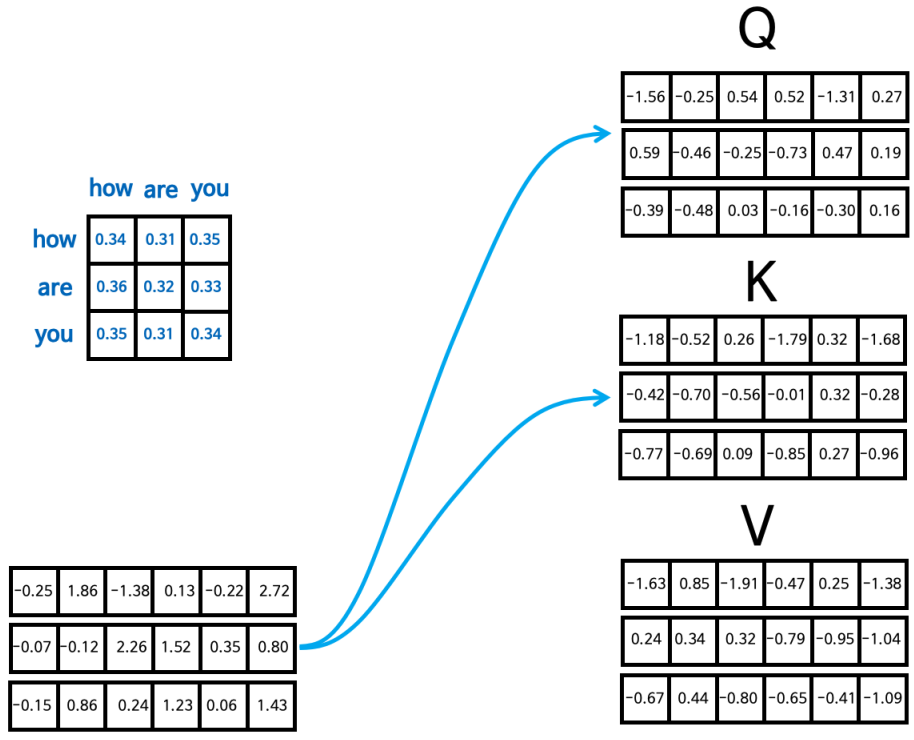
Thus, word pairs with higher relevance receive higher attention values, while those with lower relevance receive smaller values — as the model learns these relationships.



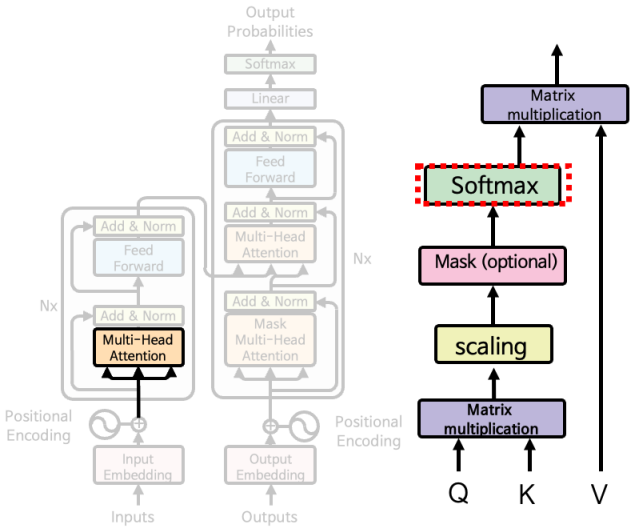
	how	are	you
how	0.34	0.31	0.35
are	0.36	0.32	0.33
you	0.35	0.31	0.34

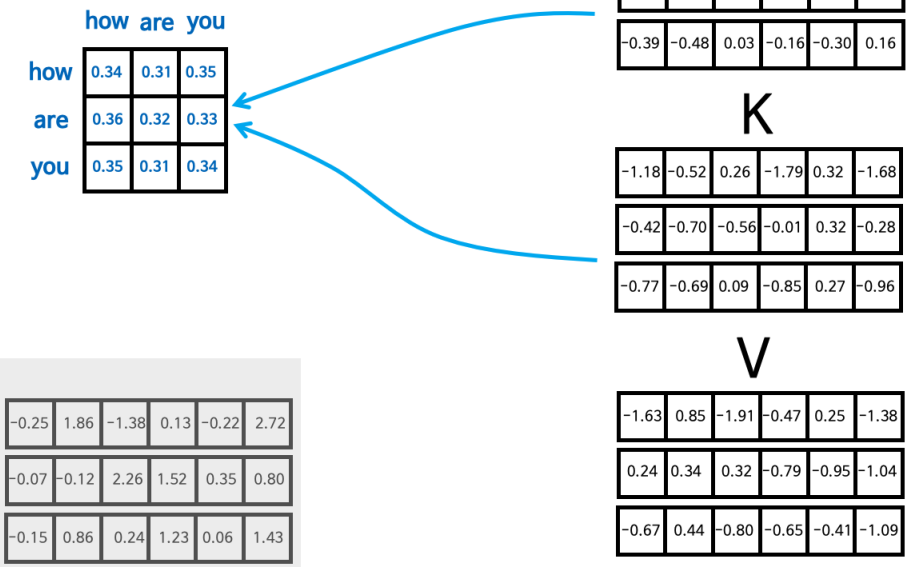
While *traditional* attention models focused on relationships between the input sequence and output sequence, self-attention takes the same input matrix and feeds it into two separate networks to produce the Q and K matrices.



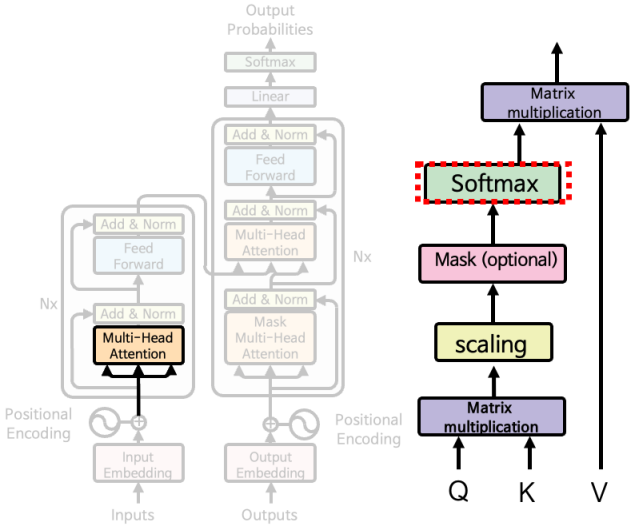


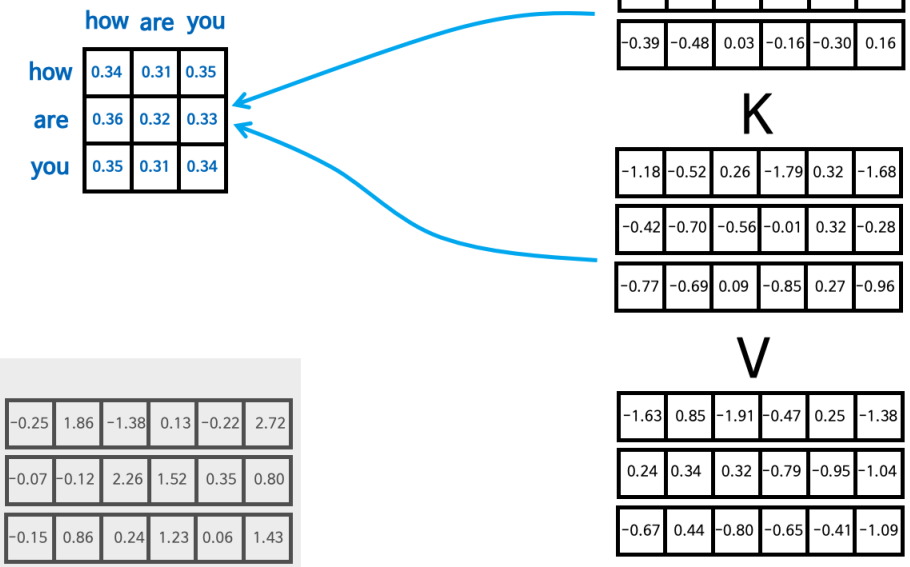
Amazingly, by simply multiplying these two matrices, the model can represent the correlations between words within a sentence.



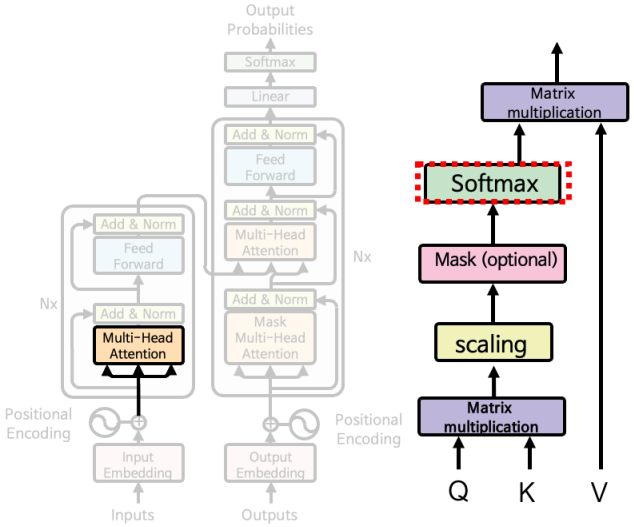


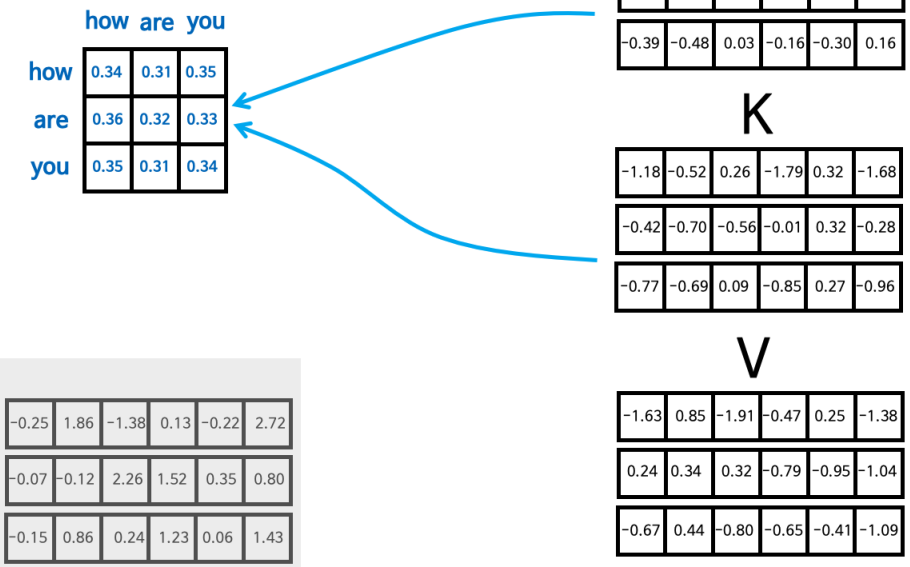
Because the model processes all words in parallel rather than sequentially, it achieves much faster computation.



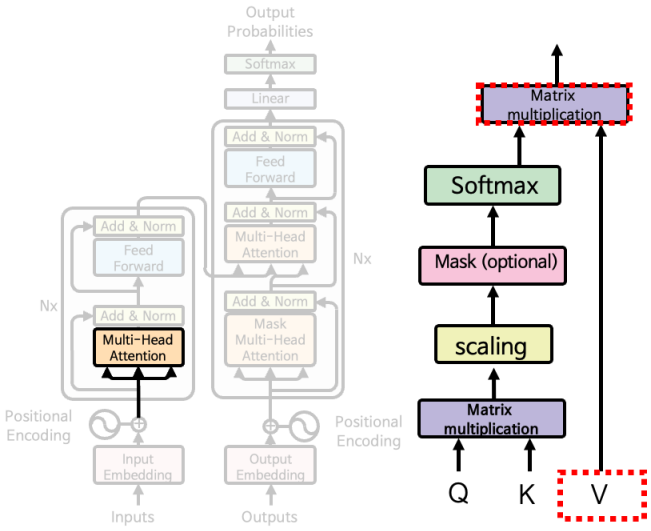


Even for long sentences, the model can calculate the attention between all pairs of words without bias or loss of information.





So, what about the final matrix multiplication?



	how	are	you
how	0.34	0.31	0.35
are	0.36	0.32	0.33
you	0.35	0.31	0.34

Q

-1.56	-0.25	0.54	0.52	-1.31	0.27
0.59	-0.46	-0.25	-0.73	0.47	0.19
-0.39	-0.48	0.03	-0.16	-0.30	0.16

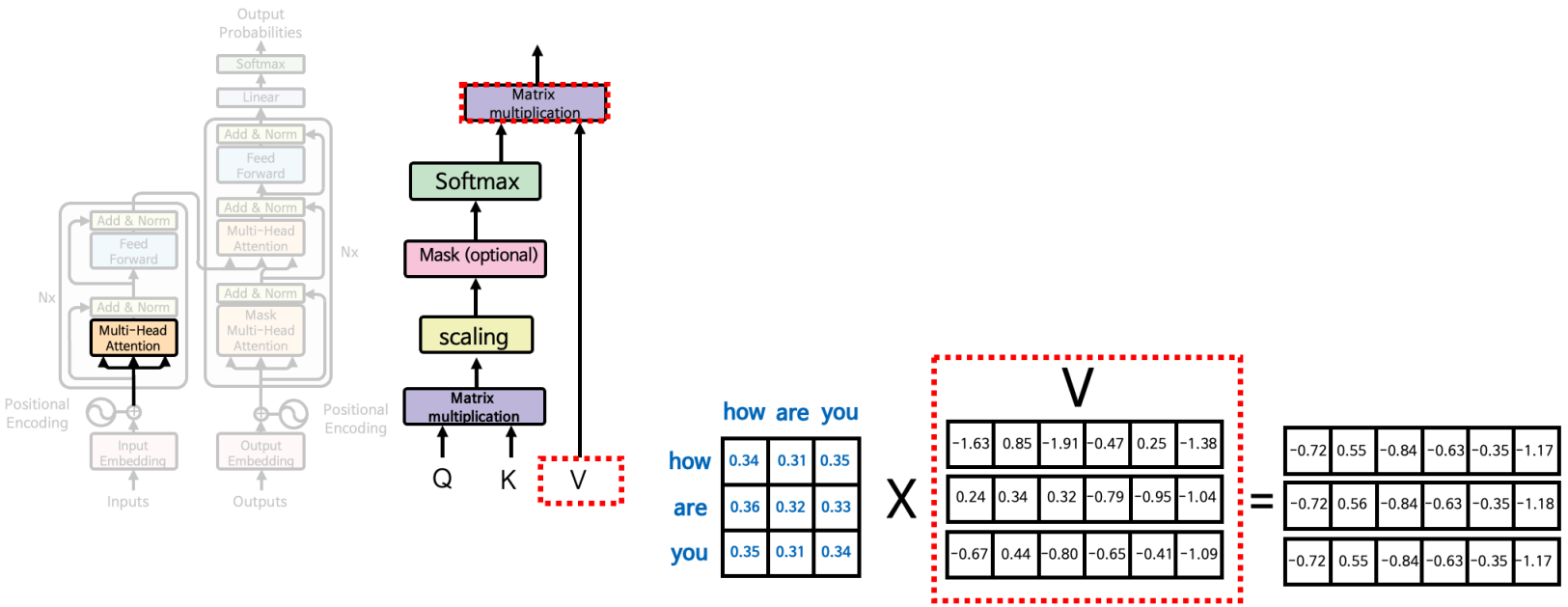
K

-1.18	-0.52	0.26	-1.79	0.32	-1.68
-0.42	-0.70	-0.56	-0.01	0.32	-0.28
-0.77	-0.69	0.09	-0.85	0.27	-0.96

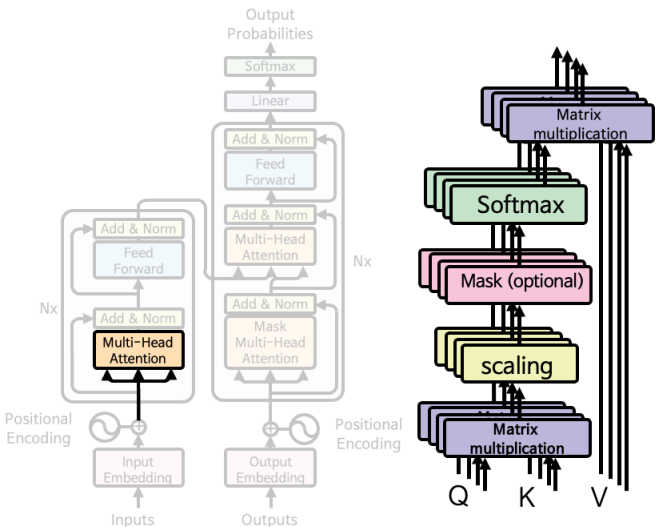
V

-1.63	0.85	-1.91	-0.47	0.25	-1.38
0.24	0.34	0.32	-0.79	-0.95	-1.04
-0.67	0.44	-0.80	-0.65	-0.41	-1.09

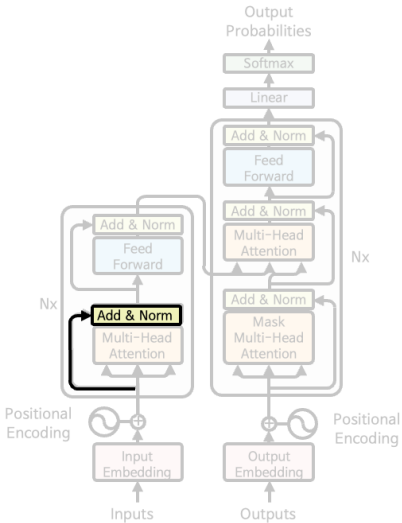
We multiply this by the V matrix to obtain the final output — a self-attended embedding that combines (1) input, (2) positional, and (3) attention information.



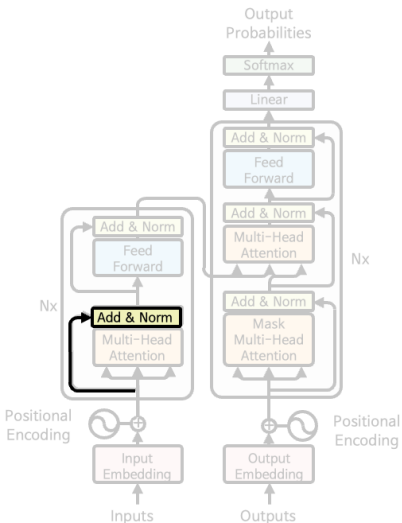
So far, we've assumed a single-head attention for simplicity, but in reality, the Transformer computes multi-head attention — the original paper uses 8 heads.



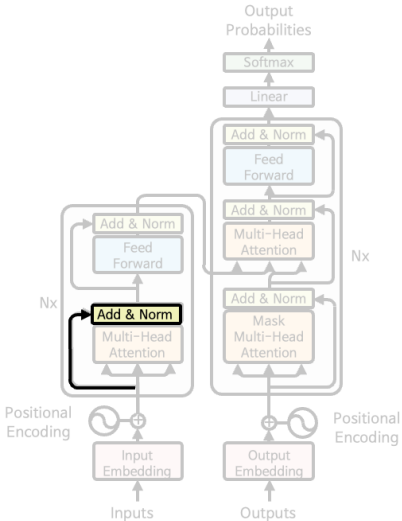
(8) We move on to the **addition and normalization** layer.



The **addition** step adds the multi-head output matrix to the initial input + positional embedding.

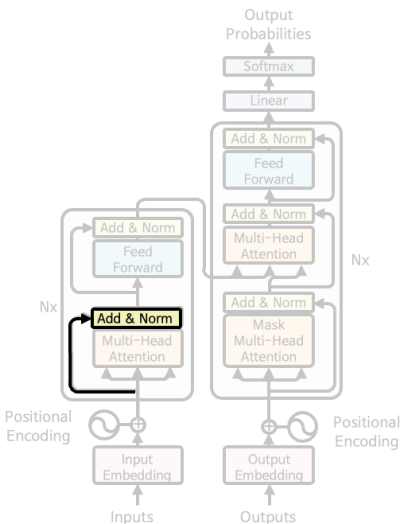


Let's assume that the resulting matrix looks like this:

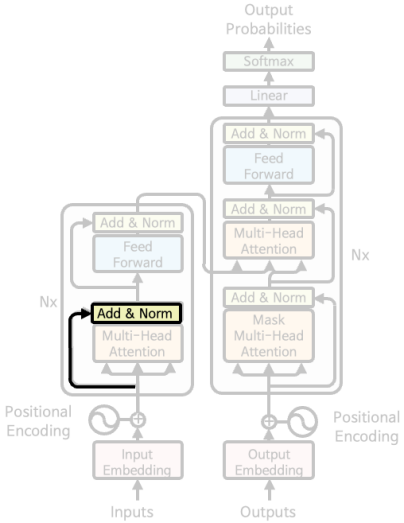


-0.59	1.99	-0.86	0.38	0.18	3.10
-0.41	0.02	2.79	1.77	0.76	1.19
-0.48	0.99	0.77	1.48	0.47	1.81

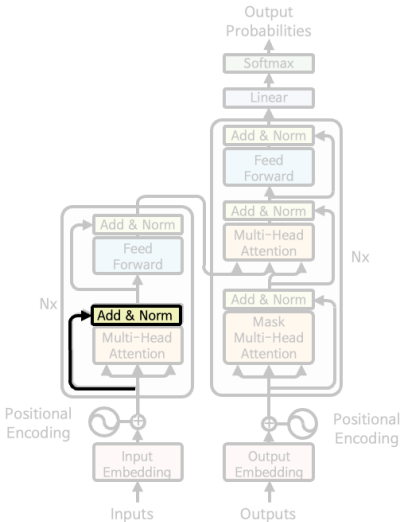
Next, for the normalization step, we first calculate the mean and standard deviation for each column.



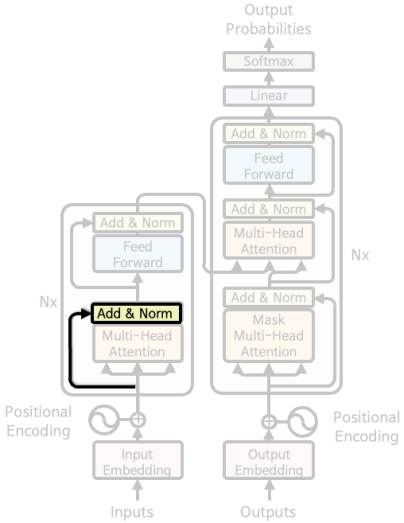
						M	SD
-0.59	1.99	-0.86	0.38	0.18	3.10	0.70	1.41
-0.41	0.02	2.79	1.77	0.76	1.19		
-0.48	0.99	0.77	1.48	0.47	1.81		



							M	SD
−0.59	1.99	−0.86	0.38	0.18	3.10	→	0.70	1.41
−0.41	0.02	2.79	1.77	0.76	1.19	→	1.02	1.07
−0.48	0.99	0.77	1.48	0.47	1.81			

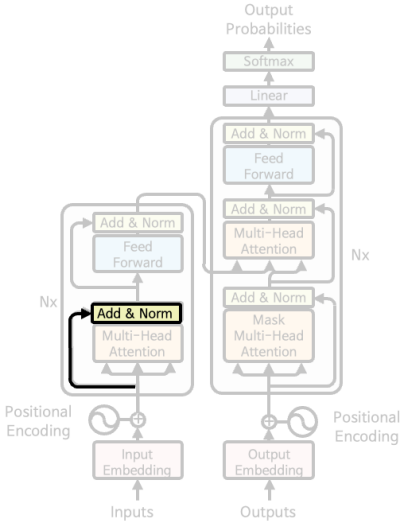


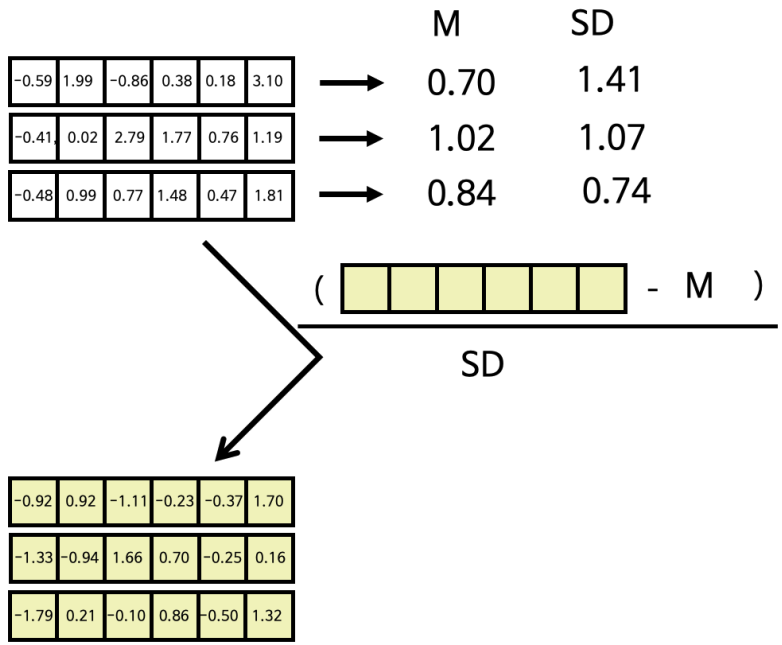
							M	SD
-0.59	1.99	-0.86	0.38	0.18	3.10	→	0.70	1.41
-0.41	0.02	2.79	1.77	0.76	1.19	→	1.02	1.07
-0.48	0.99	0.77	1.48	0.47	1.81	→	0.84	0.74



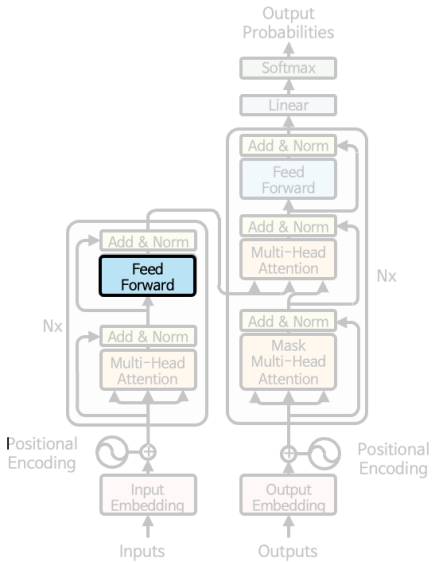
							M	SD
-0.59	1.99	-0.86	0.38	0.18	3.10	→	0.70	1.41
-0.41	0.02	2.79	1.77	0.76	1.19	→	1.02	1.07
-0.48	0.99	0.77	1.48	0.47	1.81	→	0.84	0.74

$$\frac{(\square\square\square\square\square\square - M)}{SD}$$





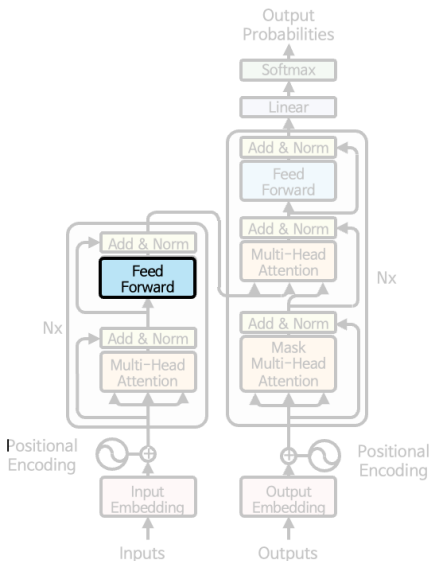
(9) Next is the **Feed-Forward** layer.



-0.92	0.92	-1.11	-0.23	-0.37	1.70
-1.33	-0.94	1.66	0.70	-0.25	0.16
-1.79	0.21	-0.10	0.86	-0.50	1.32

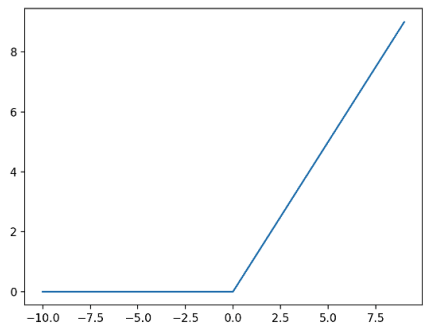
The Feed-Forward layer is a simple neural network consisting of **two layers** and using **ReLU()** as the activation function.

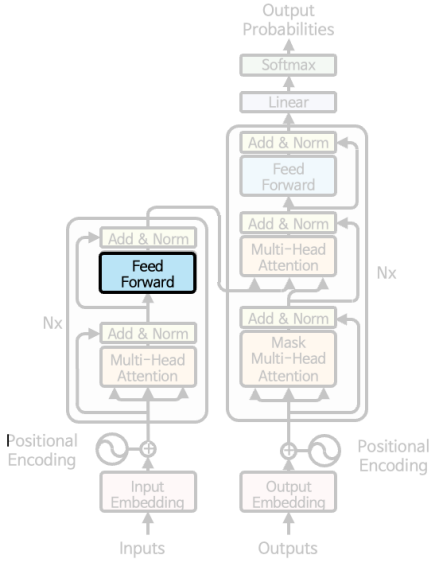
$$\mathrm{FFN}(x) = \mathrm{ReLU}(xW_1 + b_1)W_2 + b_2$$



-0.92	0.92	-1.11	-0.23	-0.37	1.70
-1.33	-0.94	1.66	0.70	-0.25	0.16
-1.79	0.21	-0.10	0.86	-0.50	1.32

ReLU function





-0.92	0.92	-1.11	-0.23	-0.37	1.70
-1.33	-0.94	1.66	0.70	-0.25	0.16
-1.79	0.21	-0.10	0.86	-0.50	1.32

X

W_1

0.31	0.20	-0.35	-0.07	-0.13	0.23
0.15	0.34	-0.38	-0.29	-0.30	-0.28
-0.28	0.21	0.17	-0.27	-0.07	0.21
-0.40	0.10	-0.20	0.14	-0.20	0.19
-0.33	-0.00	-0.08	-0.12	-0.12	0.29
0.30	-0.31	-0.23	0.25	-0.39	-0.31

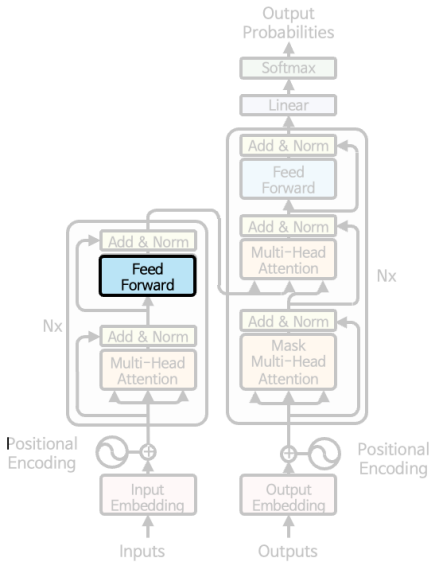
+

b_1

0.29	-0.19	0.15	0.38	-0.06	-0.00

1.20	-0.85	-0.39	0.93	-0.70	-1.38
-0.89	-0.40	1.10	0.47	0.11	0.31
0.02	-0.81	0.24	0.99	-0.50	-0.89

Applying the ReLU activation function...



-0.92	0.92	-1.11	-0.23	-0.37	1.70
-1.33	-0.94	1.66	0.70	-0.25	0.16
-1.79	0.21	-0.10	0.86	-0.50	1.32

X

w_1

0.31	0.20	-0.35	-0.07	-0.13	0.23
0.15	0.34	-0.38	-0.29	-0.30	-0.28
-0.28	0.21	0.17	-0.27	-0.07	0.21
-0.40	0.10	-0.20	0.14	-0.20	0.19
-0.33	-0.00	-0.08	-0.12	-0.12	0.29
0.30	-0.31	-0.23	0.25	-0.39	-0.31

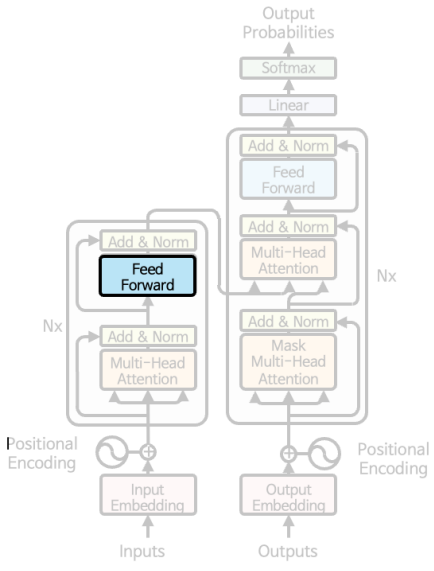
+

b_1

0.29	-0.19	0.15	0.38	-0.06	-0.00

$$ReLU\left(\begin{array}{|c|c|c|c|c|c|}\hline 1.20 & -0.85 & -0.39 & 0.93 & -0.70 & -1.38 \\ \hline -0.89 & -0.40 & 1.10 & 0.47 & 0.11 & 0.31 \\ \hline 0.02 & -0.81 & 0.24 & 0.99 & -0.50 & -0.89 \\ \hline \end{array}\right)$$

Applying the ReLU activation function...



-0.92	0.92	-1.11	-0.23	-0.37	1.70
-1.33	-0.94	1.66	0.70	-0.25	0.16
-1.79	0.21	-0.10	0.86	-0.50	1.32

X

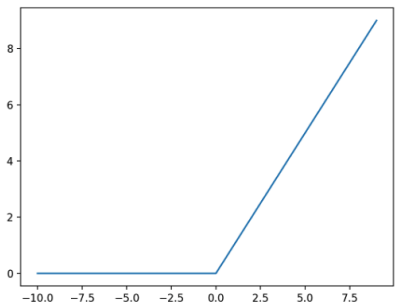
w_1

0.31	0.20	-0.35	-0.07	-0.13	0.23
0.15	0.34	-0.38	-0.29	-0.30	-0.28
-0.28	0.21	0.17	-0.27	-0.07	0.21
-0.40	0.10	-0.20	0.14	-0.20	0.19
-0.33	-0.00	-0.08	-0.12	-0.12	0.29
0.30	-0.31	-0.23	0.25	-0.39	-0.31

+

b_1

0.29	-0.19	0.15	0.38	-0.06	-0.00

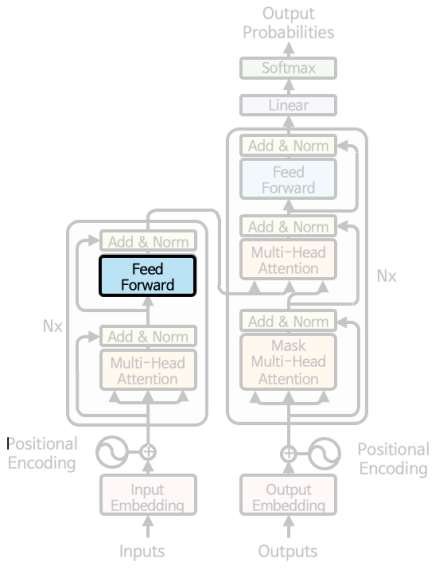


$ReLU($

1.20	-0.85	-0.39	0.93	-0.70	-1.38
-0.89	-0.40	1.10	0.47	0.11	0.31
0.02	-0.81	0.24	0.99	-0.50	-0.89

$)$

Negative values become 0.



-0.92	0.92	-1.11	-0.23	-0.37	1.70
-1.33	-0.94	1.66	0.70	-0.25	0.16
-1.79	0.21	-0.10	0.86	-0.50	1.32

X

W_1

0.31	0.20	-0.35	-0.07	-0.13	0.23
0.15	0.34	-0.38	-0.29	-0.30	-0.28
-0.28	0.21	0.17	-0.27	-0.07	0.21
-0.40	0.10	-0.20	0.14	-0.20	0.19
-0.33	-0.00	-0.08	-0.12	-0.12	0.29
0.30	-0.31	-0.23	0.25	-0.39	-0.31

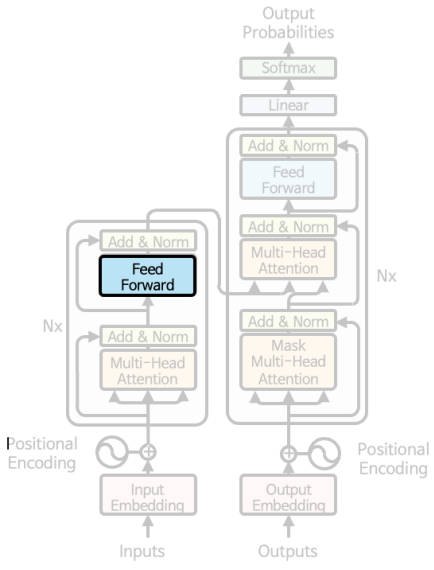
+

b_1

0.29	-0.19	0.15	0.38	-0.06	-0.00

1.20	0.00	0.00	0.93	0.00	0.00
0.00	0.00	1.10	0.47	0.11	0.31
0.02	0.00	0.24	0.99	0.00	0.00

(10) Then, by multiplying with the second layer's weights and biases, we obtain the final output of the Feed-Forward layer.



1.20	0.00	0.00	0.93	0.00	0.00
0.00	0.00	1.10	0.47	0.11	0.31
0.02	0.00	0.24	0.99	0.00	0.00

X

W_2

-0.09	0.39	-0.09	-0.39	-0.37	-0.14
-0.34	-0.10	-0.24	0.33	-0.40	-0.13
0.20	-0.34	-0.04	0.34	0.15	0.01
-0.41	0.09	-0.20	-0.06	-0.22	-0.09
0.25	0.23	-0.17	-0.05	-0.26	-0.14
0.31	-0.41	-0.13	-0.17	-0.03	-0.20

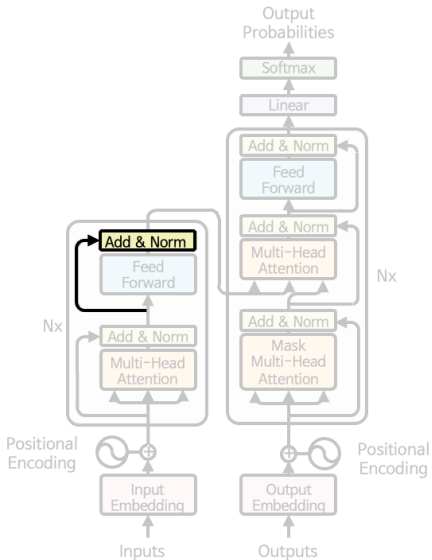
+

b_2

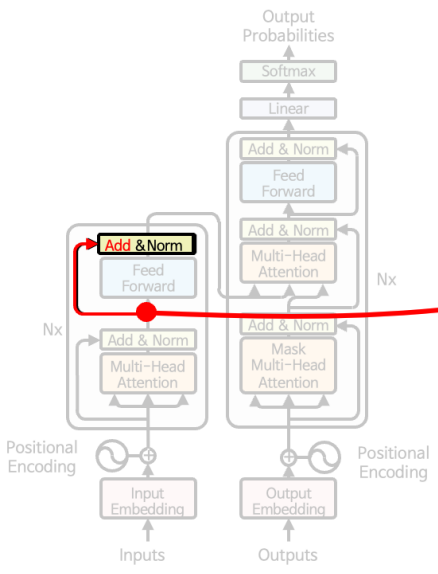
-0.33	0.34	-0.16	0.11	-0.14	0.03

-0.82	0.89	-0.46	-0.42	-0.79	-0.21
-0.19	-0.08	-0.36	0.40	-0.12	-0.07
-0.69	0.36	-0.37	0.12	-0.34	-0.05

(11) We repeat the same **Add & Normalize** process.



As before, we first calculate the sum of the two matrices.



Feed-Forward Layer Output Matrix

-0.82	0.89	-0.46	-0.42	-0.79	-0.21
-0.19	-0.08	-0.36	0.40	-0.12	-0.07
-0.69	0.36	-0.37	0.12	-0.34	-0.05

+

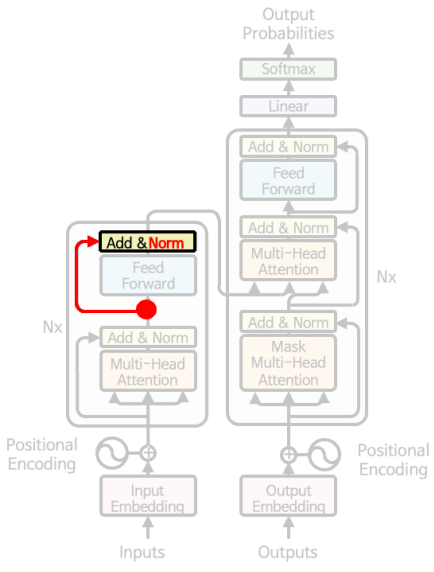
1st Addition & Normalization

-0.92	0.92	-1.11	-0.23	-0.37	1.70
-1.33	-0.94	1.66	0.70	-0.25	0.16
-1.79	0.21	-0.10	0.86	-0.50	1.32

=

-1.74	1.81	-1.57	-0.65	-1.16	1.49
-1.52	-1.02	1.30	1.10	-0.37	0.09
-2.48	0.57	-0.47	0.98	-0.84	1.27

Then, we normalize the matrix again:



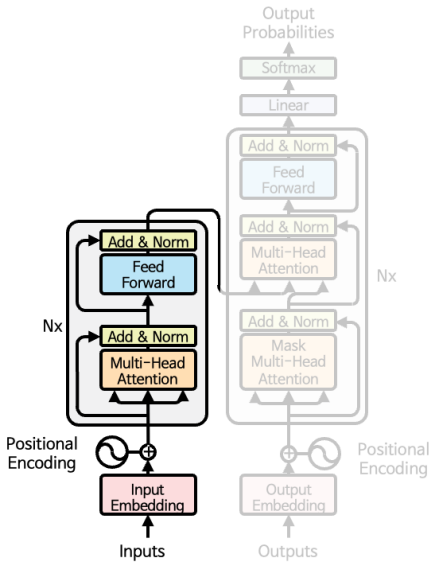
-1.74	1.81	-1.57	-0.65	-1.16	1.49
-1.52	-1.02	1.30	1.10	-0.37	0.09
-2.48	0.57	-0.47	0.98	-0.84	1.27

M	SD
-0.30	1.43
-0.07	1.03
-0.16	1.28

$$\frac{(\square\square\square\square\square\square - M)}{SD}$$

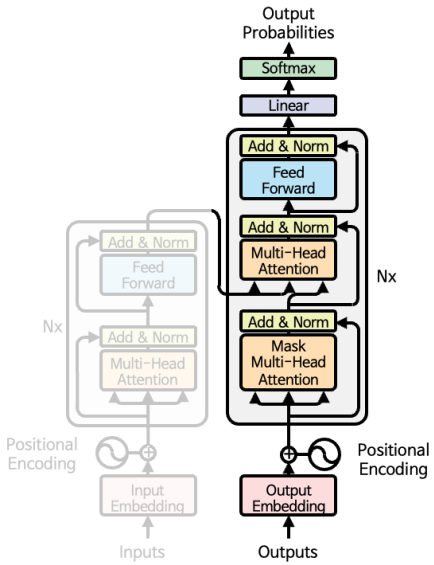
-1.01	1.48	-0.89	-0.24	-0.60	1.26
-1.41	-0.92	1.33	1.13	-0.29	0.15
-1.81	0.57	-0.24	0.89	-0.53	1.12

This resulting matrix is the **final output of the encoder.**

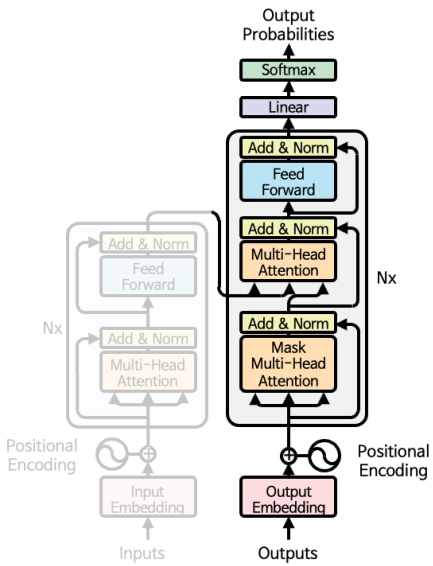


-1.01	1.48	-0.89	-0.24	-0.60	1.26
-1.41	-0.92	1.33	1.13	-0.29	0.15
-1.81	0.57	-0.24	0.89	-0.53	1.12

Now, let's move on to the **Decoder**.



(1) We performs **word encoding** and **positional encoding** of the output (target) sequence.

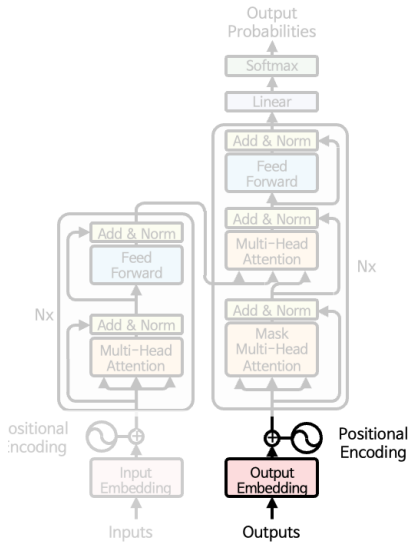


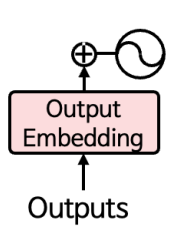
Decoder input: [⟨sos⟩, I, am] → Index: [0, 6, 3]

Decoder output target : [I, am, fine] → Index : [6, 3, 4]

The process of word encoding and positional encoding is identical to that in the encoder. In fact, the positional encoding values can be reused from the encoder.

$$PE_{(pos,2i)} = sin(\frac{pos}{10000^{\frac{2i}{d_{model}}}}) \quad PE_{(pos,2i+1)} = cos(\frac{pos}{10000^{\frac{2i}{d_{model}}}})$$



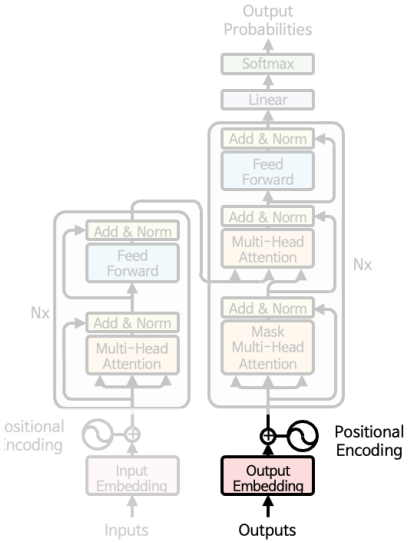


	0	1	2	3	4	5	
	0	1	0	1	0	1	0 pos
	0.84	0.54	0.05	1	0	1	1 pos
	0.91	-0.42	0.09	1	0	1	2 pos

6

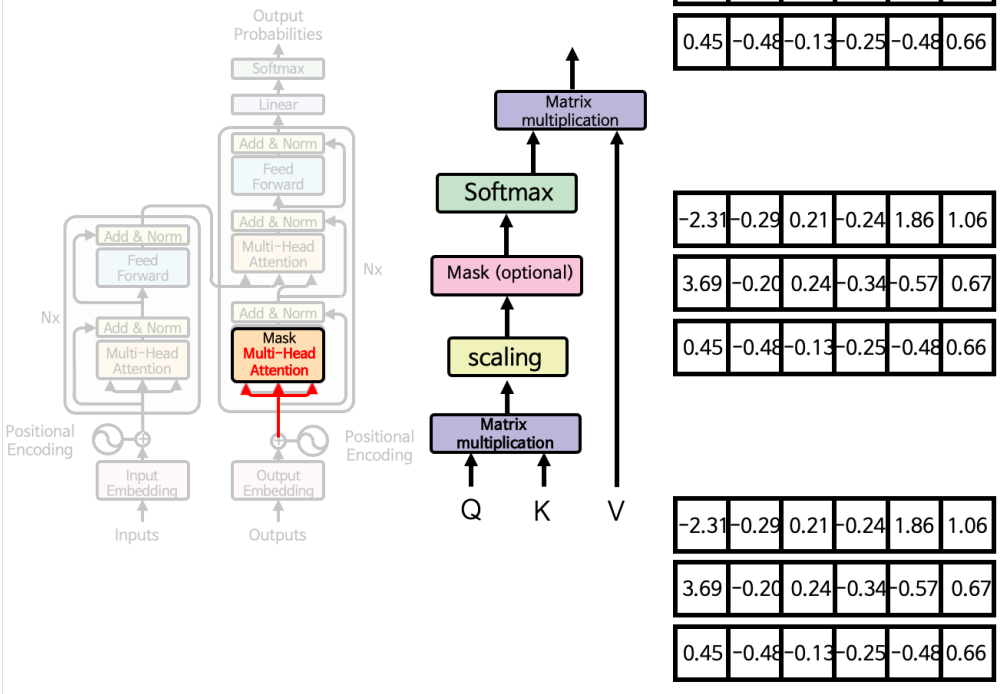
	6					
0, ⟨sos⟩	-2.31	-1.29	0.21	-1.24	1.86	0.06
6, I	2.85	-0.74	0.20	-1.34	-0.57	-0.33
3, am	-0.46	-0.06	-0.22	-1.25	-0.49	-0.34

Here's how it looks:

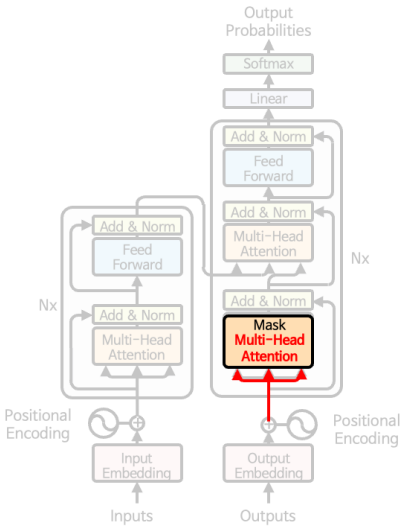


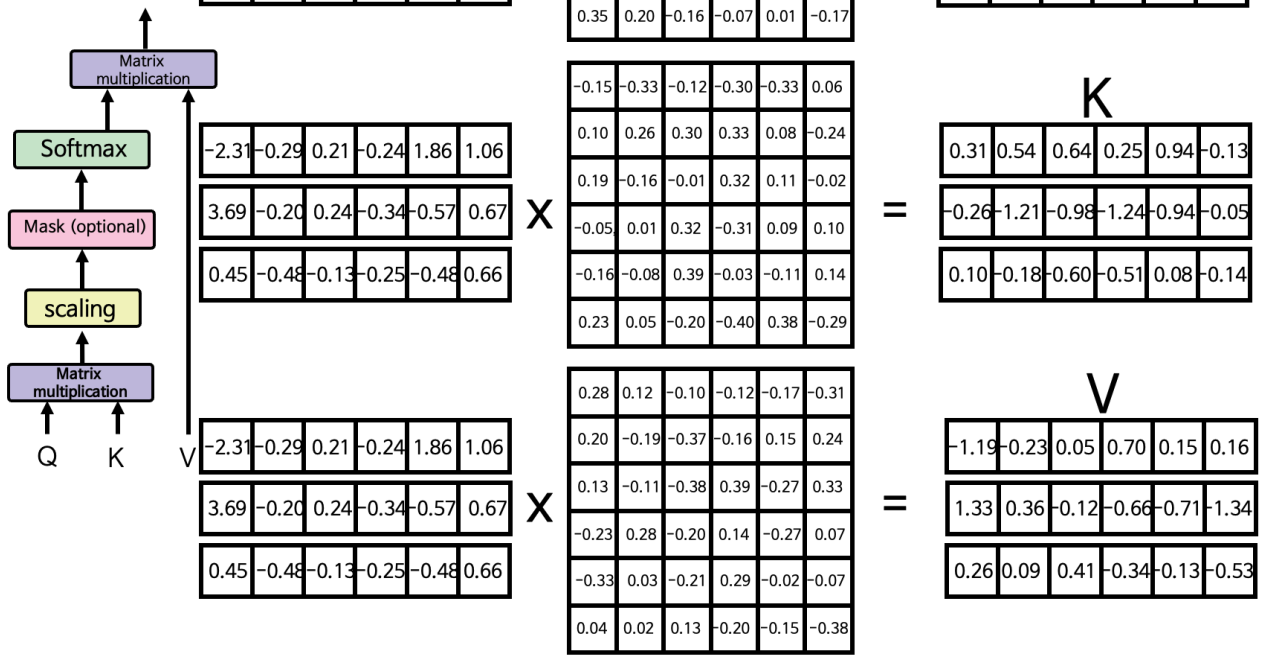
0	-2.31	-1.29	0.21	-1.24	1.86	0.06
0.	2.85	-0.74	0.20	-1.34	-0.57	-0.33
0.	-0.46	-0.06	-0.22	-1.25	-0.49	-0.34

(2) The **Masked Multi-Head Attention** operation in the decoder is almost the same as in the encoder.

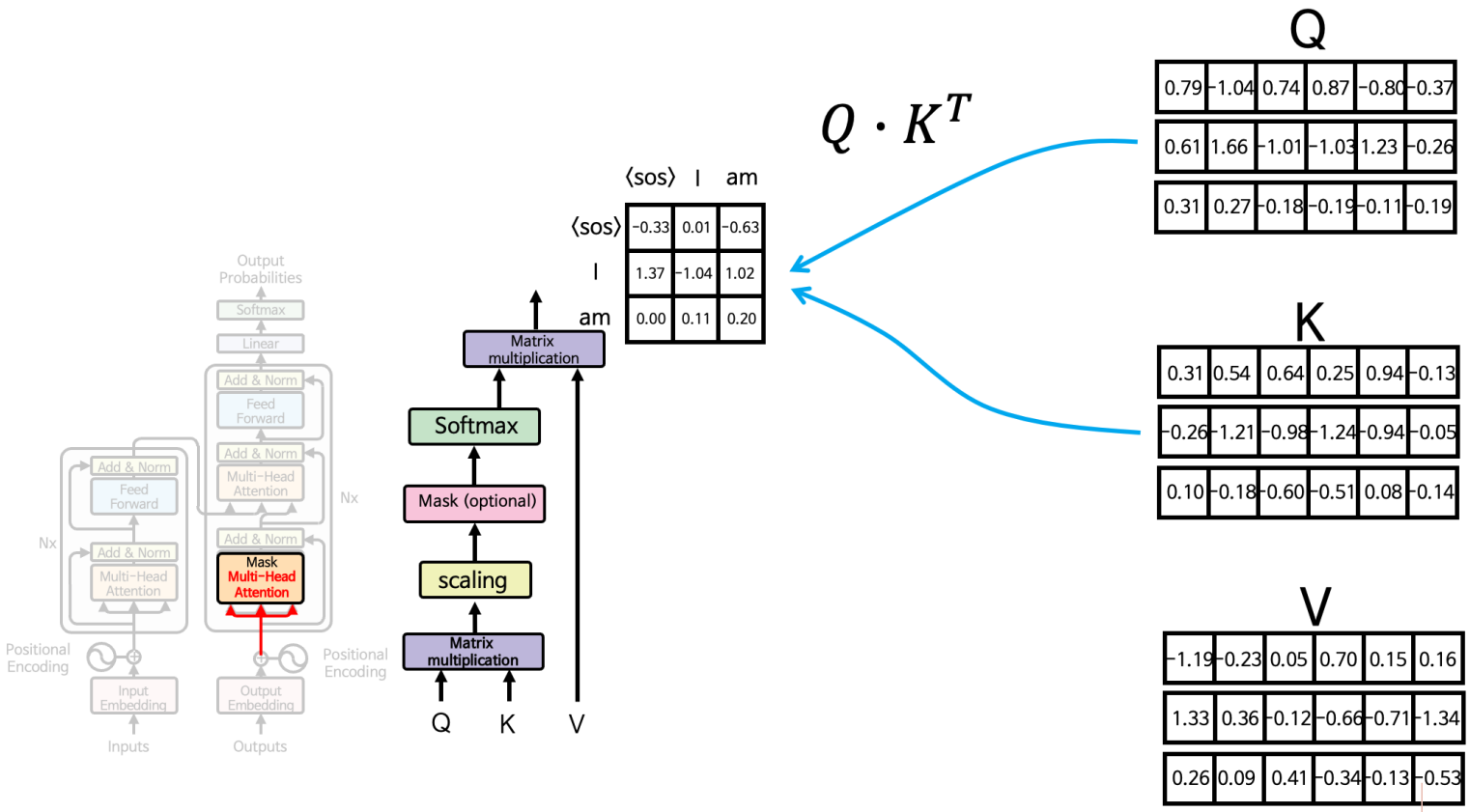


The **Masked Multi-Head Attention** operation in the decoder is almost the same as in the encoder.

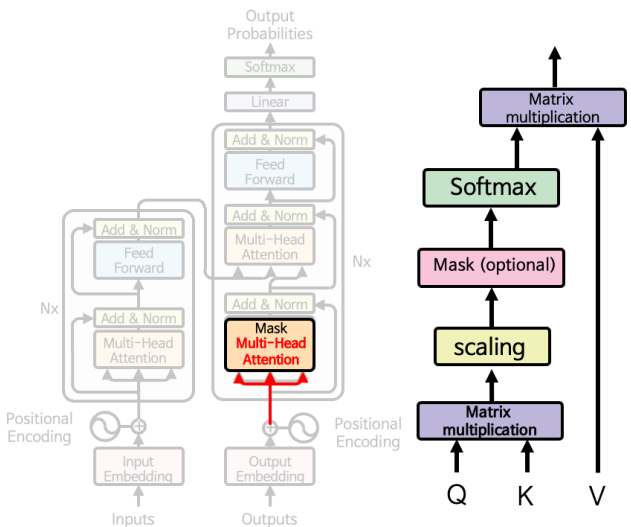




We multiply the Q and K matrices to create the attention matrix.



We then scale the matrix by dividing it by $\sqrt{6}$, just as before, so that the range of values changes accordingly.

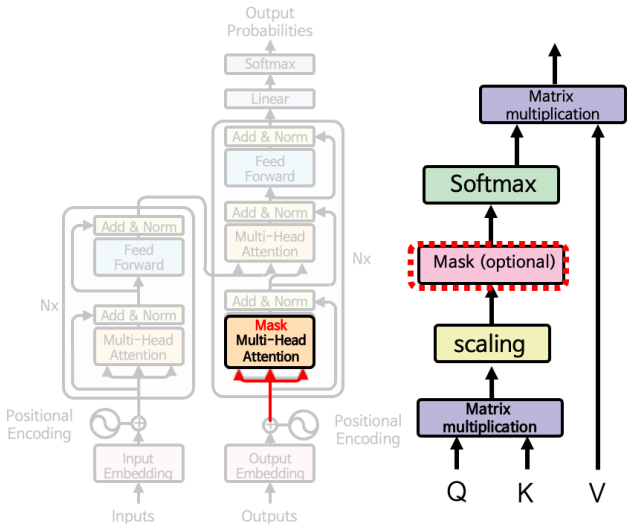


	⟨sos⟩	I	am
⟨sos⟩	-0.33	0.01	-0.63
I	1.37	-1.04	1.02
am	0.00	0.11	0.20

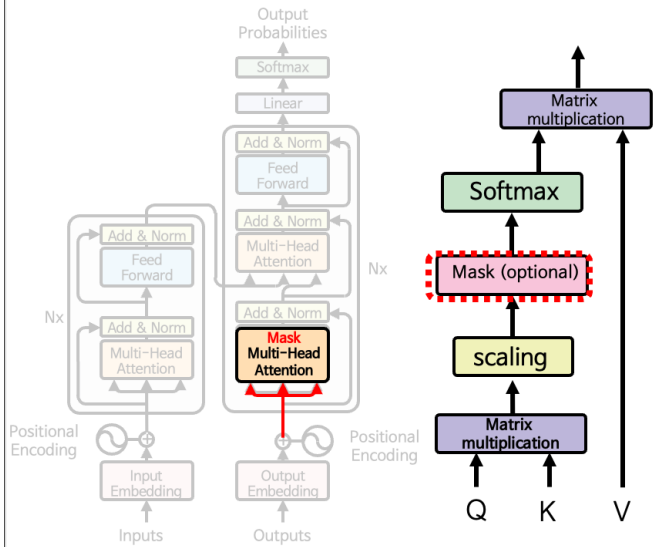
$\div \sqrt{6} =$

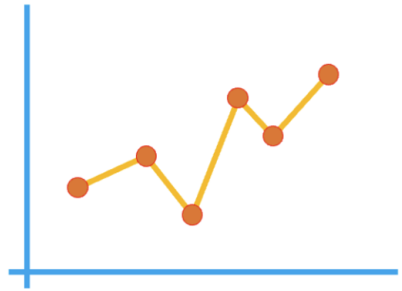
-0.13	0.00	-0.26
0.56	-0.43	0.42
0.00	0.04	0.08

(3) Let's explore the key concept of the decoder's multi-head attention — **masking**.



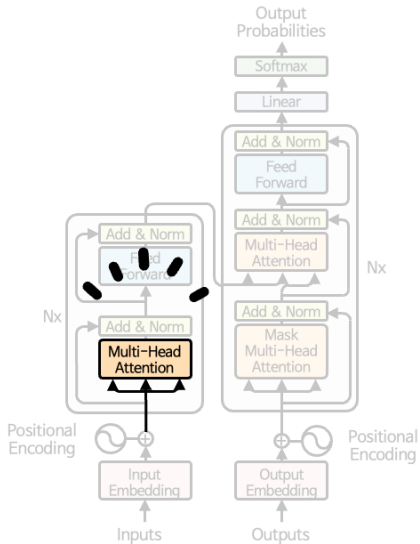
The goal of the transformer decoder is to generate the output word sequence, one token at a time (recall: language modeling).

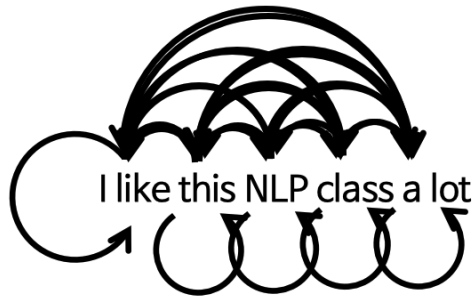




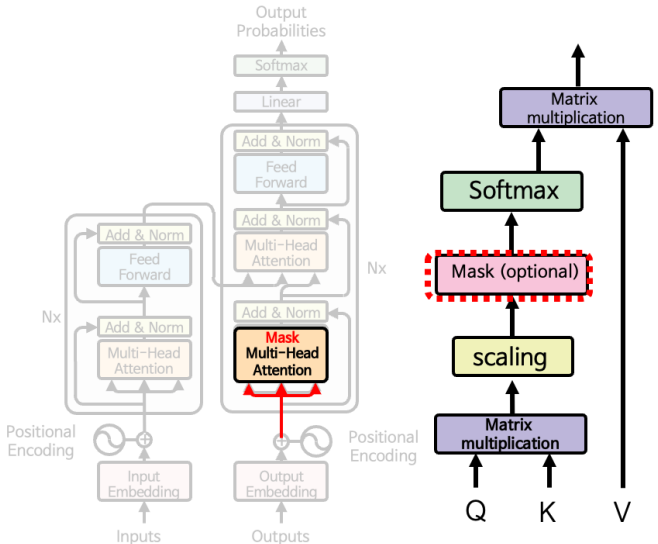
This is an awesome sentence that was written

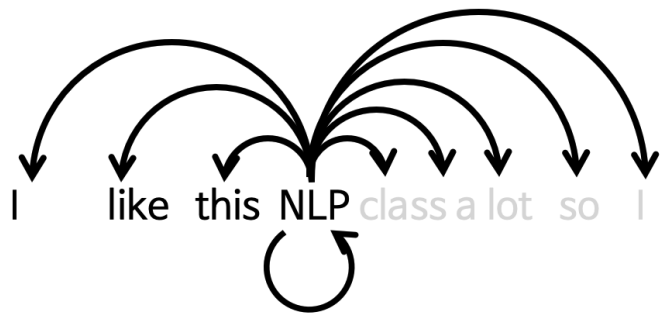
While the encoder needs to consider all tokens in the input sentence to understand the full meaning,



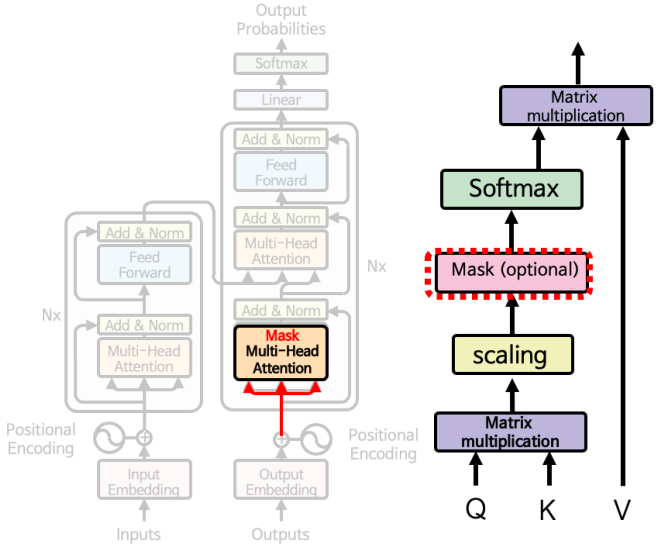


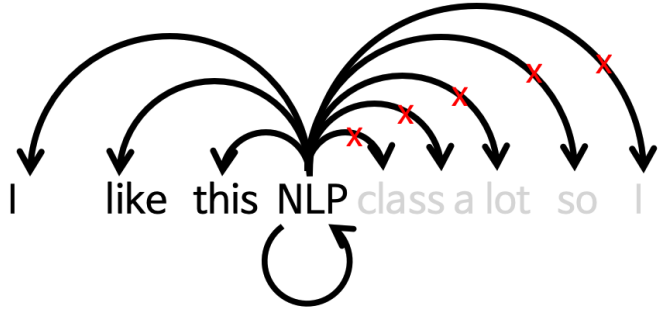
the decoder generates output **one word at a time**.



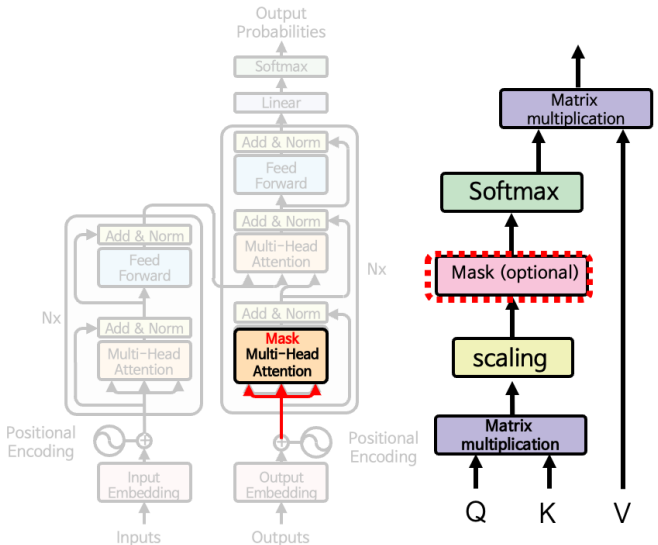


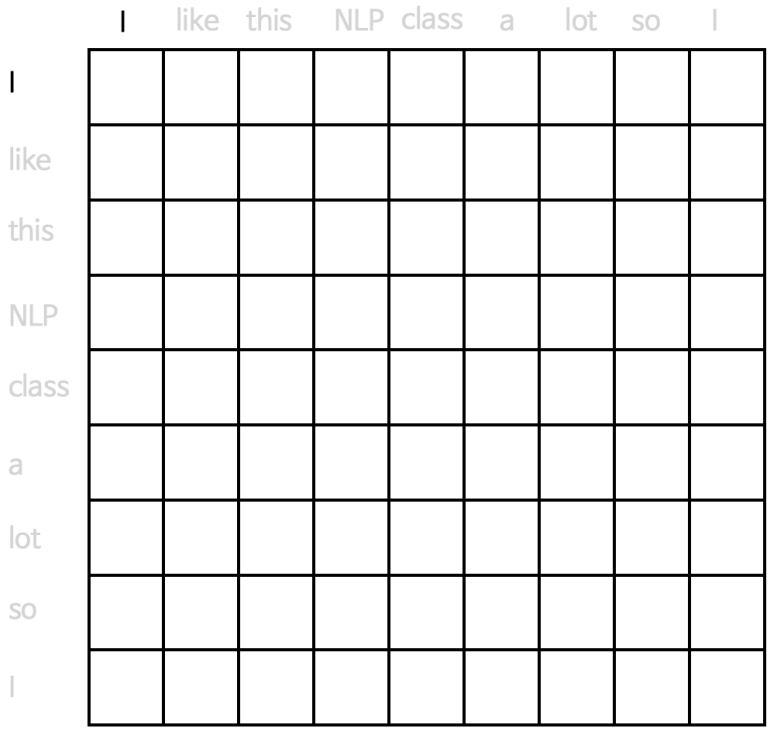
Therefore, it's natural that the decoder should NOT attend to words that haven't been generated yet.



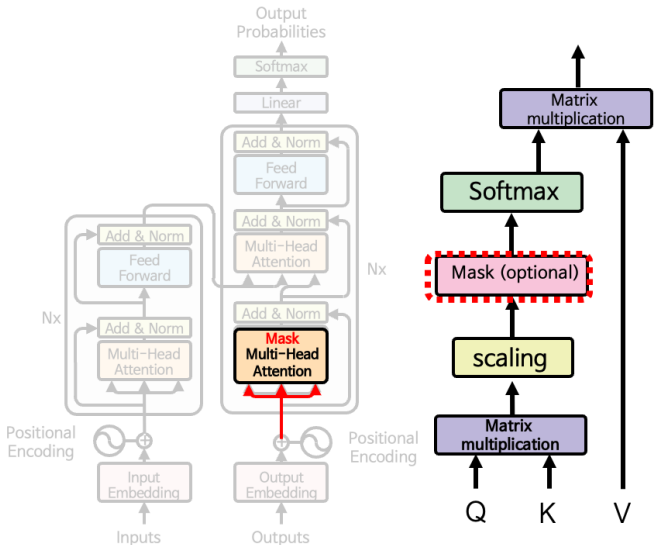


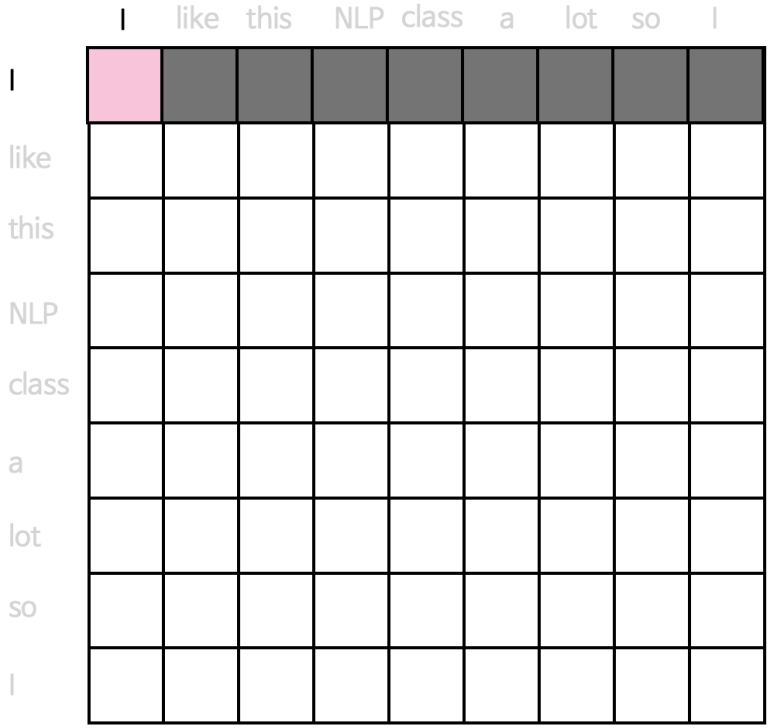
To reflect this characteristic, the decoder applies a masking mechanism during training.

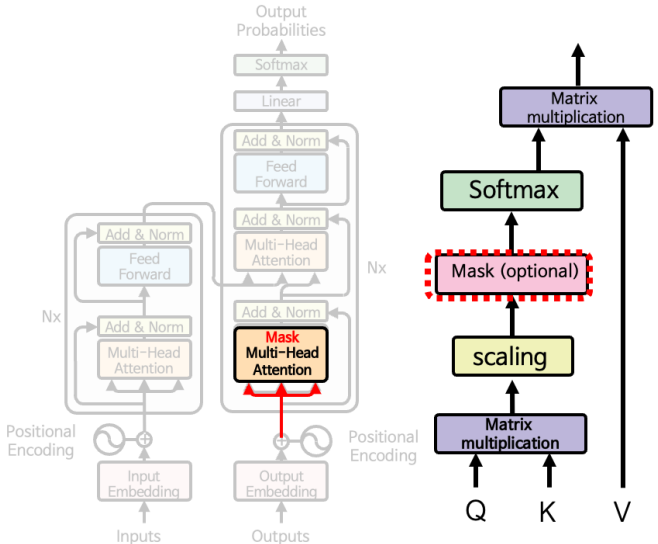


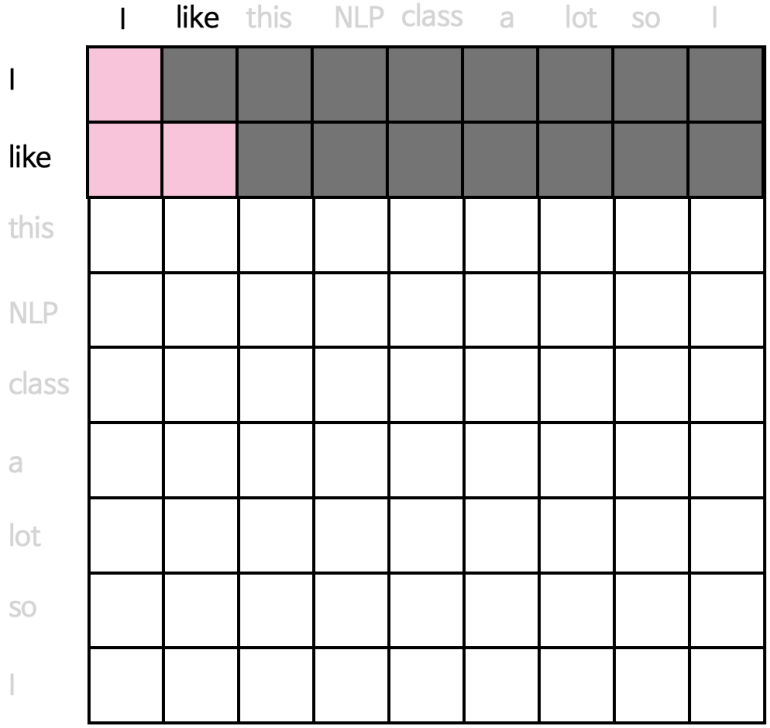


The key idea is to hide future tokens so they do not affect the current prediction.






Output Probabilities
Softmax
Linear
Add & Norm
Feed Forward
Add & Norm
Multi-Head Attention
Add & Norm
Mask Multi-Head Attention
Nx
Add & Norm
Feed Forward
Add & Norm
Multi-Head Attention
Nx
Positional Encoding
Positional Encoding
Input Embedding
Output Embedding
Inputs
Outputs
Matrix multiplication
Softmax
Mask (optional)
scaling
Matrix multiplication
Q
K
V


I
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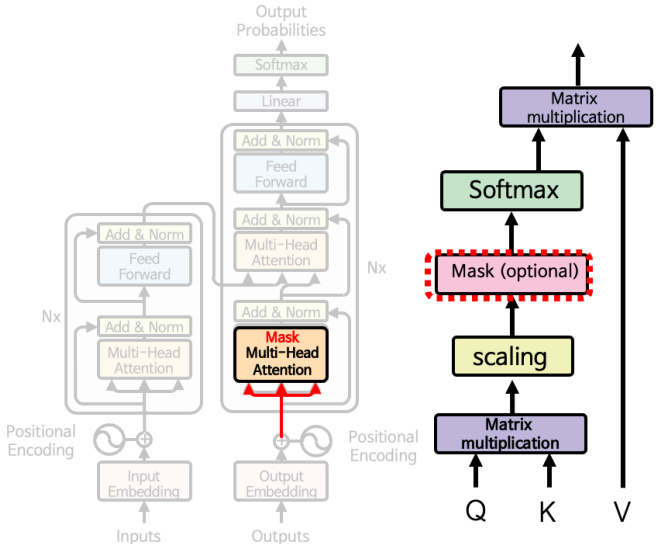
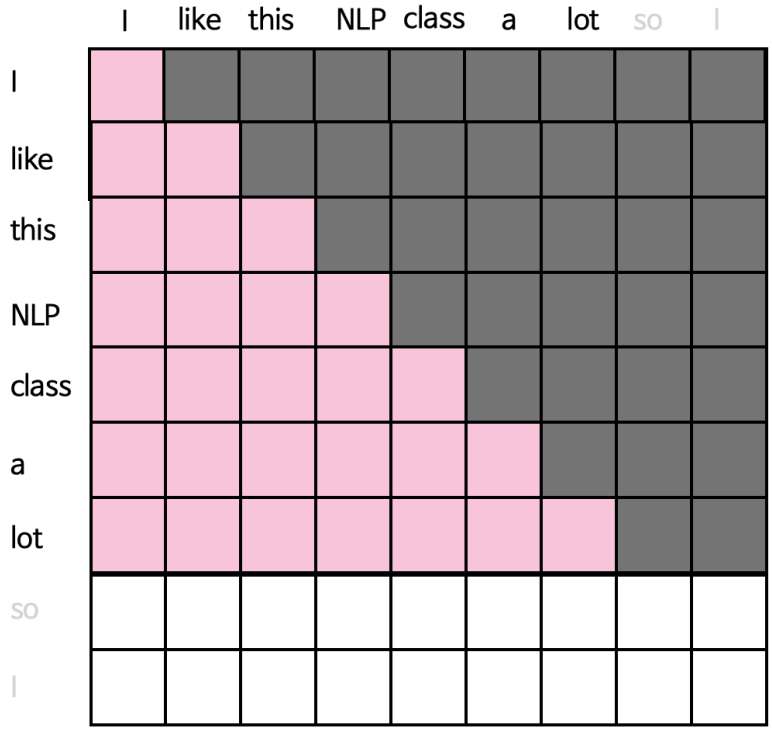

Output Probabilities
Softmax
Linear
Add & Norm
Feed Forward
Add & Norm
Multi-Head Attention
Add & Norm
Mask Multi-Head Attention
Nx
Add & Norm
Feed Forward
Add & Norm
Multi-Head Attention
Nx
Positional Encoding
Positional Encoding
Input Embedding
Output Embedding
Inputs
Outputs
Matrix multiplication
Softmax
Mask (optional)
scaling
Matrix multiplication
Q
K
V


I
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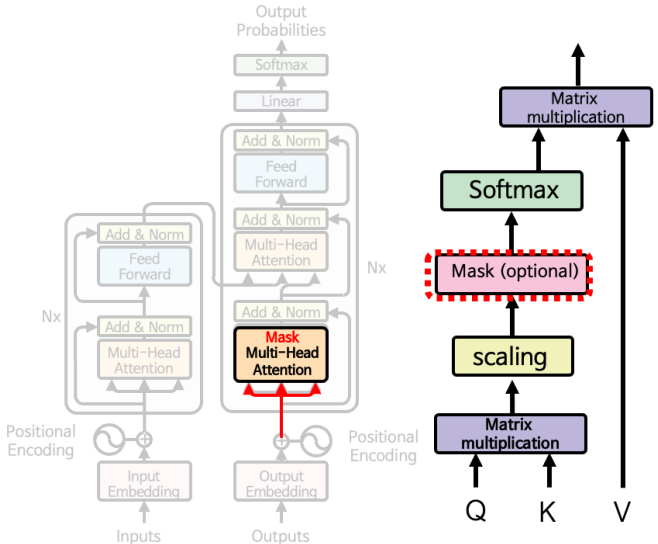
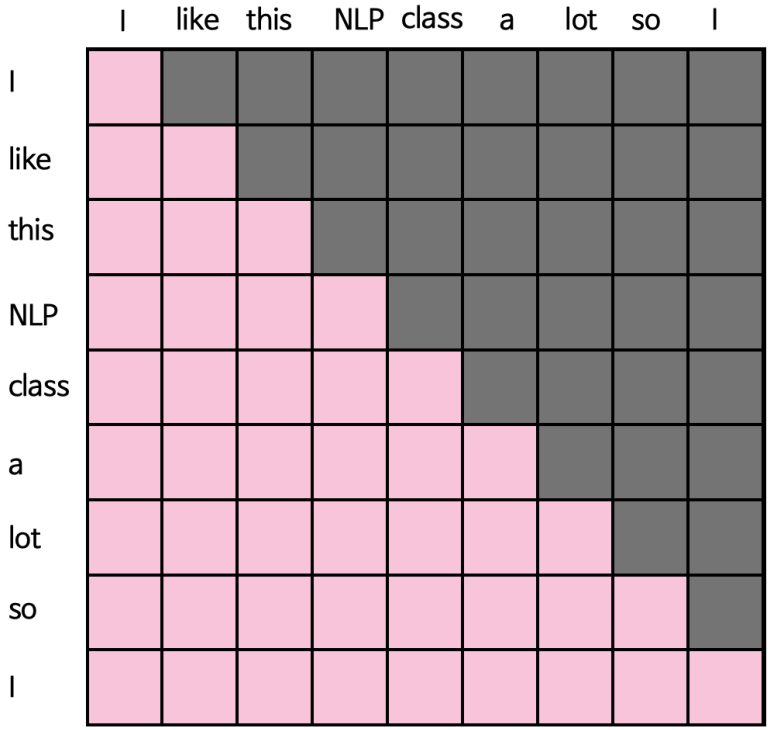

Output Probabilities
Softmax
Linear
Add & Norm
Feed Forward
Add & Norm
Multi-Head Attention
Add & Norm
Mask Multi-Head Attention
Nx
Add & Norm
Feed Forward
Add & Norm
Multi-Head Attention
Nx
Positional Encoding
Positional Encoding
Input Embedding
Output Embedding
Inputs
Outputs
Matrix multiplication
Softmax
Mask (optional)
scaling
Matrix multiplication
Q
K
V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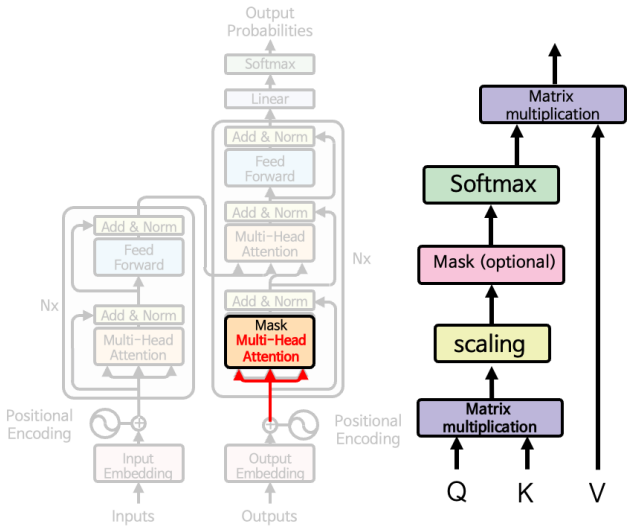
When this masking algorithm is applied to the attention matrix, we get:

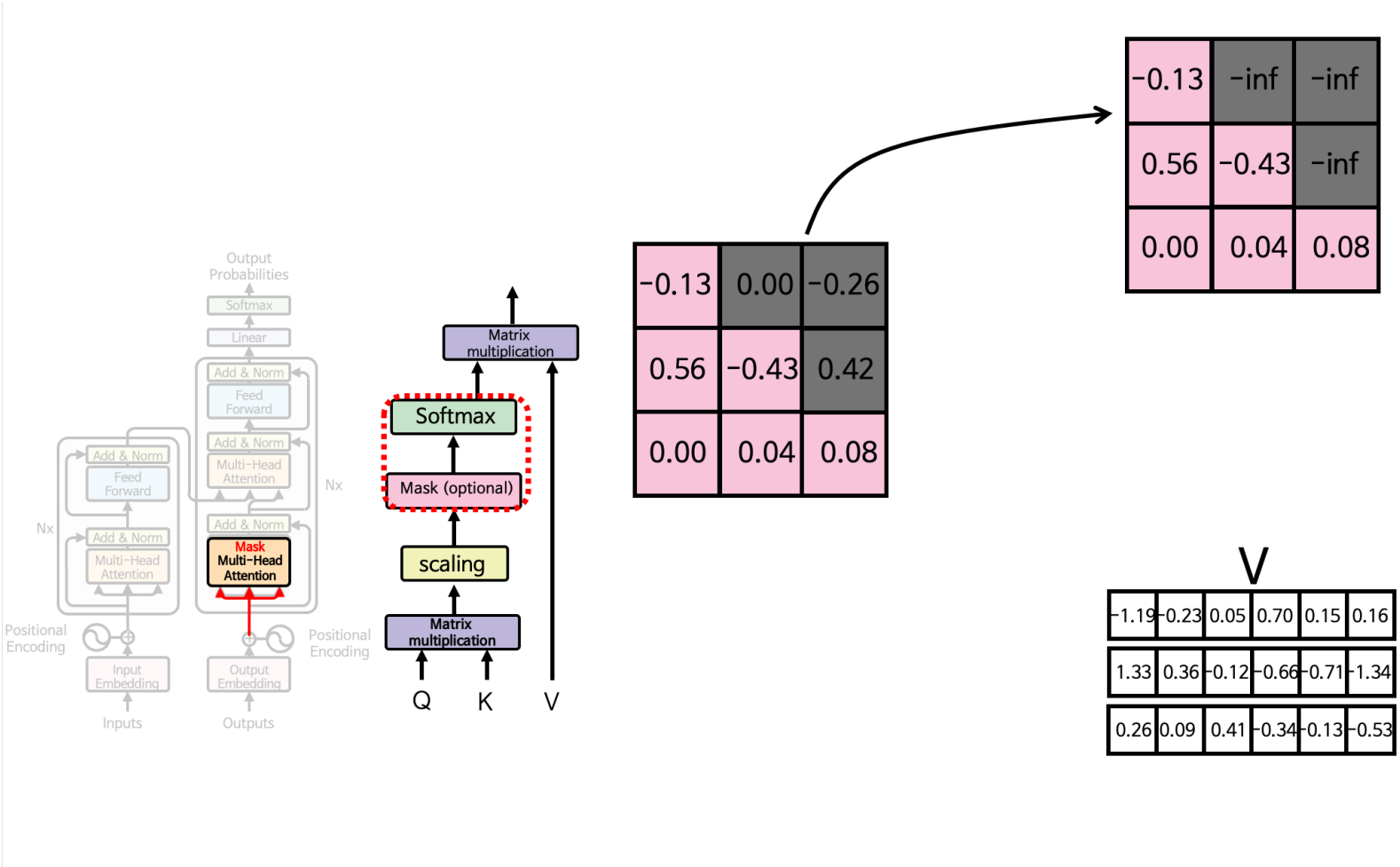


	⟨sos⟩	I	am
⟨sos⟩	-0.33	0.01	-0.63
I	1.37	-1.04	1.02
am	0.00	0.11	0.20

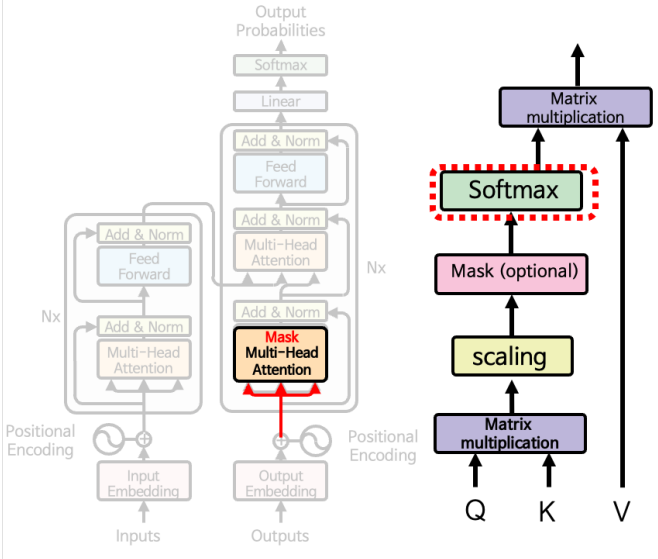
$\div \sqrt{6} =$

-0.13	0.00	-0.26
0.56	-0.43	0.42
0.00	0.04	0.08

We add *–inf* to the masked positions because, after passing through the softmax layer, *–inf* becomes 0, effectively eliminating attention to those positions.



Feeding this matrix into the softmax layer gives us:



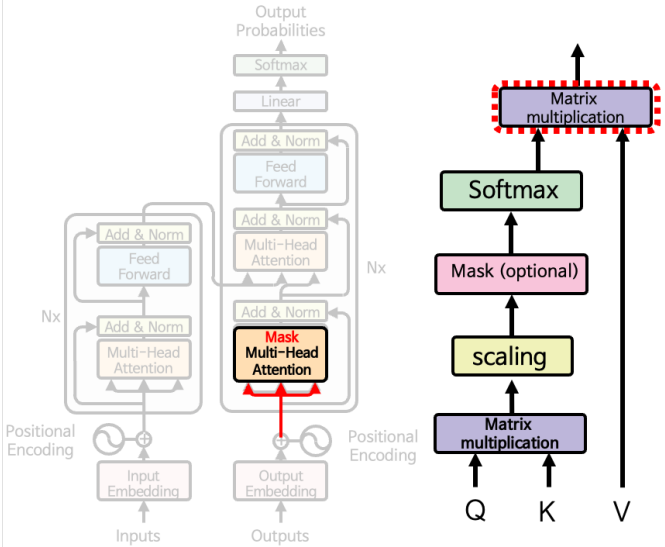
-0.13	0.00	-0.26
0.56	-0.43	0.42
0.00	0.04	0.08

1	0	0
0.73	0.27	0
0.32	0.33	0.35

V

-1.19	-0.23	0.05	0.70	0.15	0.16
1.33	0.36	-0.12	-0.66	-0.71	-1.34
0.26	0.09	0.41	-0.34	-0.13	-0.53

Then we multiply the two matrices as follows:



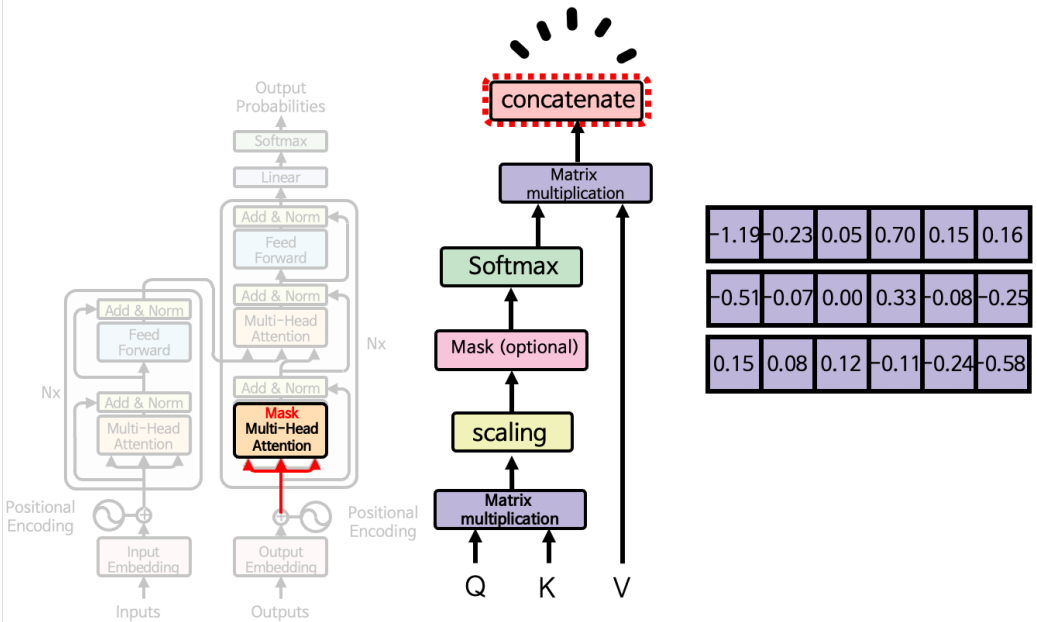
-1.19	-0.23	0.05	0.70	0.15	0.16
-0.51	-0.07	0.00	0.33	-0.08	-0.25
0.15	0.08	0.12	-0.11	-0.24	-0.58

1	0	0
0.73	0.27	0
0.32	0.33	0.35

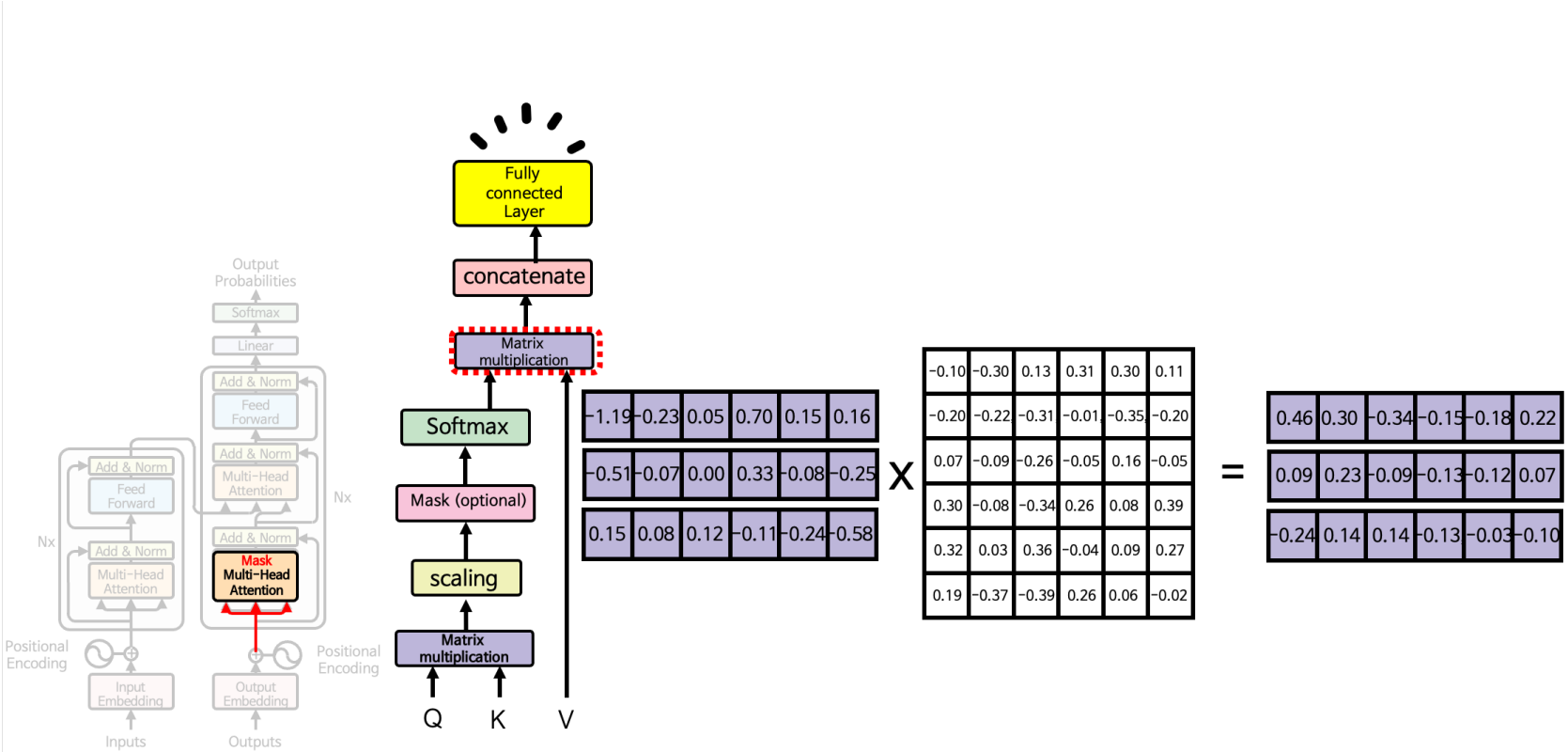
V

-1.19	-0.23	0.05	0.70	0.15	0.16
1.33	0.36	-0.12	-0.66	-0.71	-1.34
0.26	0.09	0.41	-0.34	-0.13	-0.53

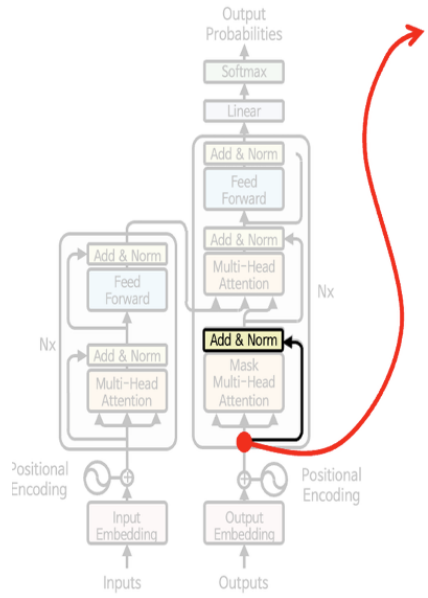
Next, we concatenate the resulting matrices (if needed):



We then multiply again to produce the final matrix of the masked multi-head attention.



(4) We apply the same Add & Normalize process again.

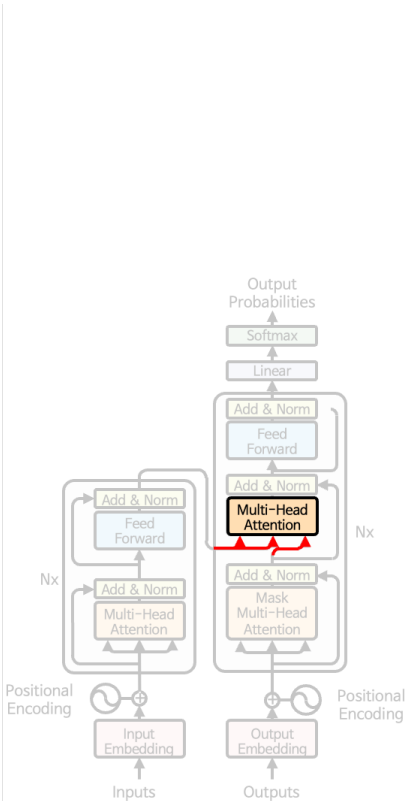


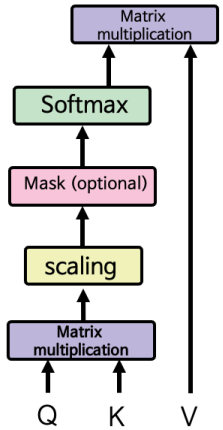
-2.31	-0.29	0.21	-0.24	1.86	1.06		0.46	0.30	-0.34	-0.15	-0.18	0.22
3.69	-0.20	0.24	-0.34	-0.57	0.67	+	0.09	0.23	-0.09	-0.13	-0.12	0.07
0.45	-0.48	-0.13	-0.25	-0.48	0.66		-0.24	0.14	0.14	-0.13	-0.03	-0.10

=

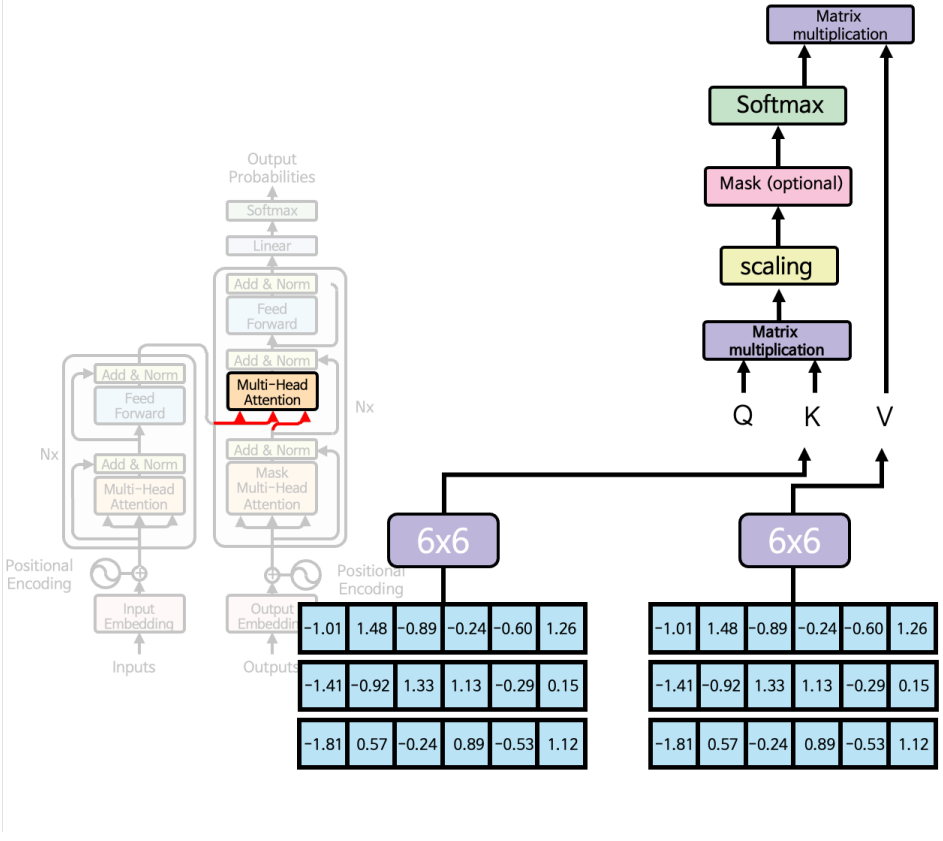
$$Norm\left(\begin{array}{cccccc} -1.85 & 0.01 & -0.12 & -0.39 & 1.67 & 1.28 \\ 3.79 & 0.03 & 0.15 & -0.47 & -0.69 & 0.74 \\ 0.21 & -0.33 & 0.01 & -0.38 & -0.51 & 0.56 \end{array}\right) = \begin{array}{cccccc} -1.69 & -0.08 & -0.20 & -0.42 & 1.37 & 1.02 \\ 2.13 & -0.37 & -0.29 & -0.71 & -0.85 & 0.10 \\ 0.76 & -0.70 & 0.23 & -0.82 & -1.16 & 1.69 \end{array}$$

(6) The decoder's second multi-head attention operates the same way as the encoder's, except for the inputs.

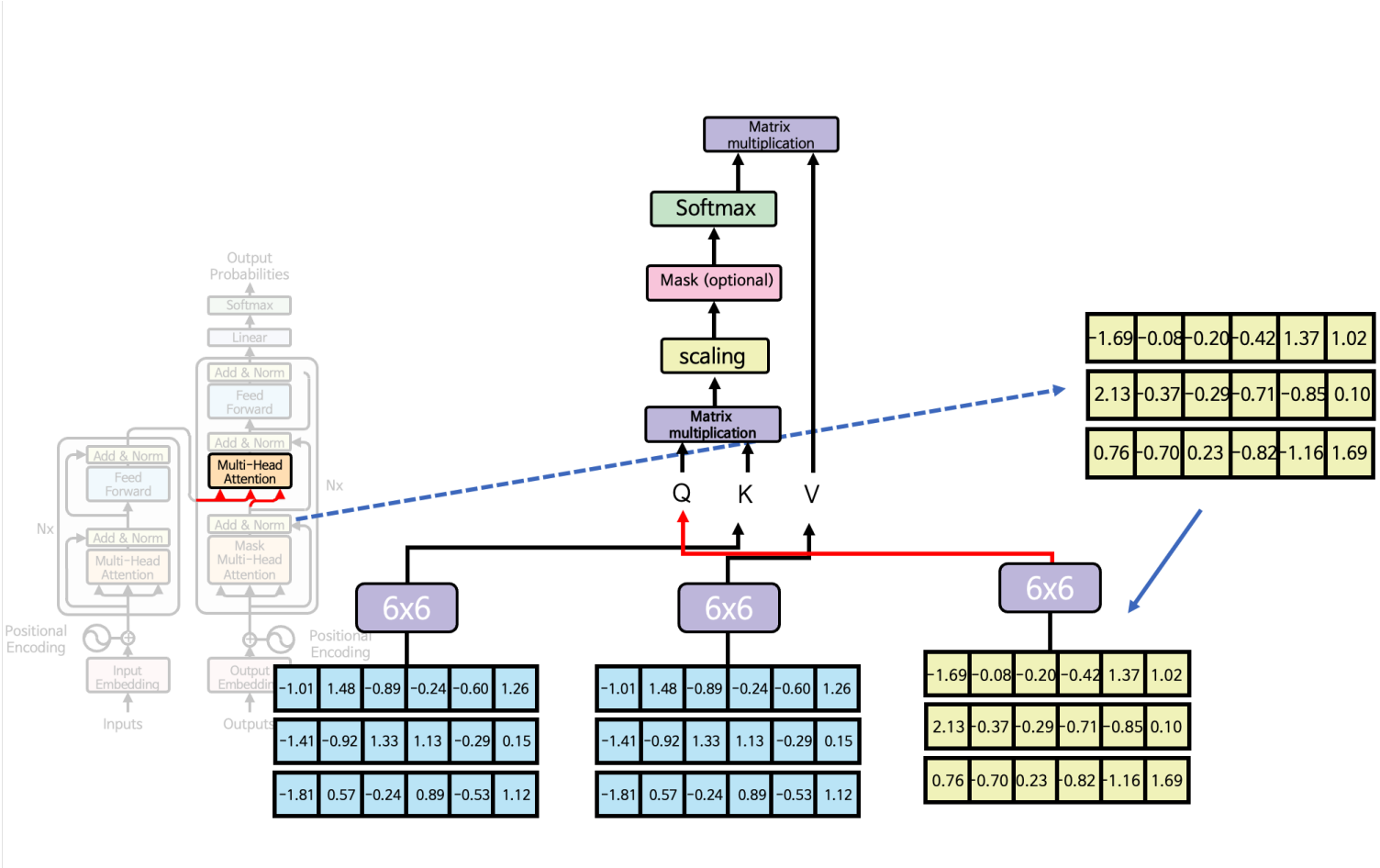




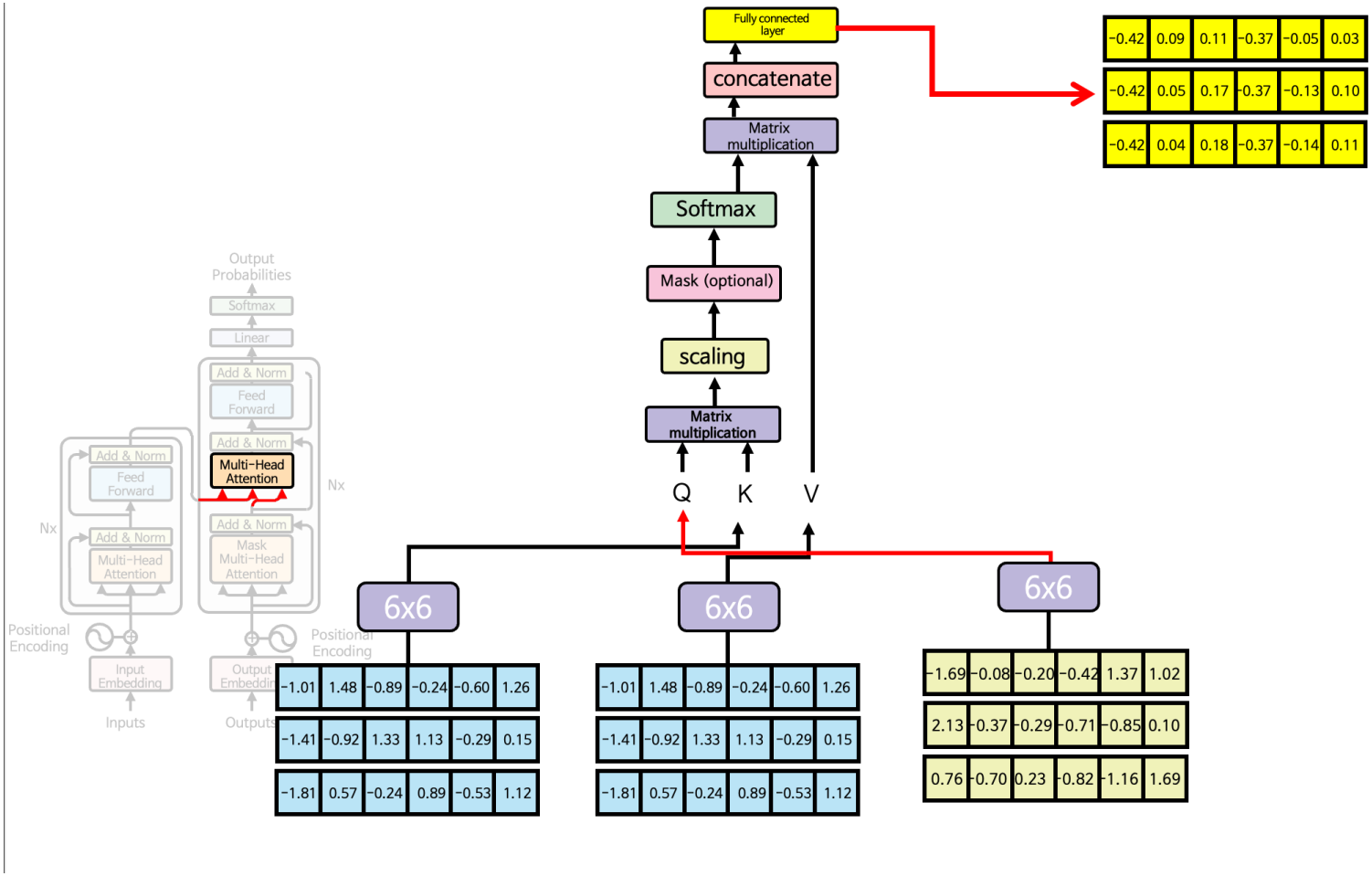
Here, the values of K and V are derived from the **encoder's final output**, multiplied by a 6×6 matrix.



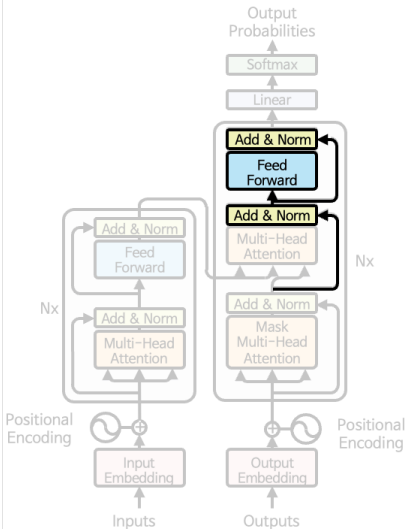
The value of Q comes from the output of the decoder's previous Add & Norm layer. **In other words, the decoder determines which parts of the encoder's output (K, V) to attend to, based on the context it has generated so far (Q).**



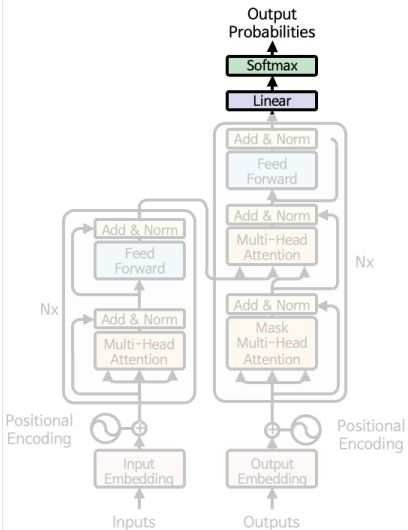
Assume the final output of this multi-head attention looks as follows:



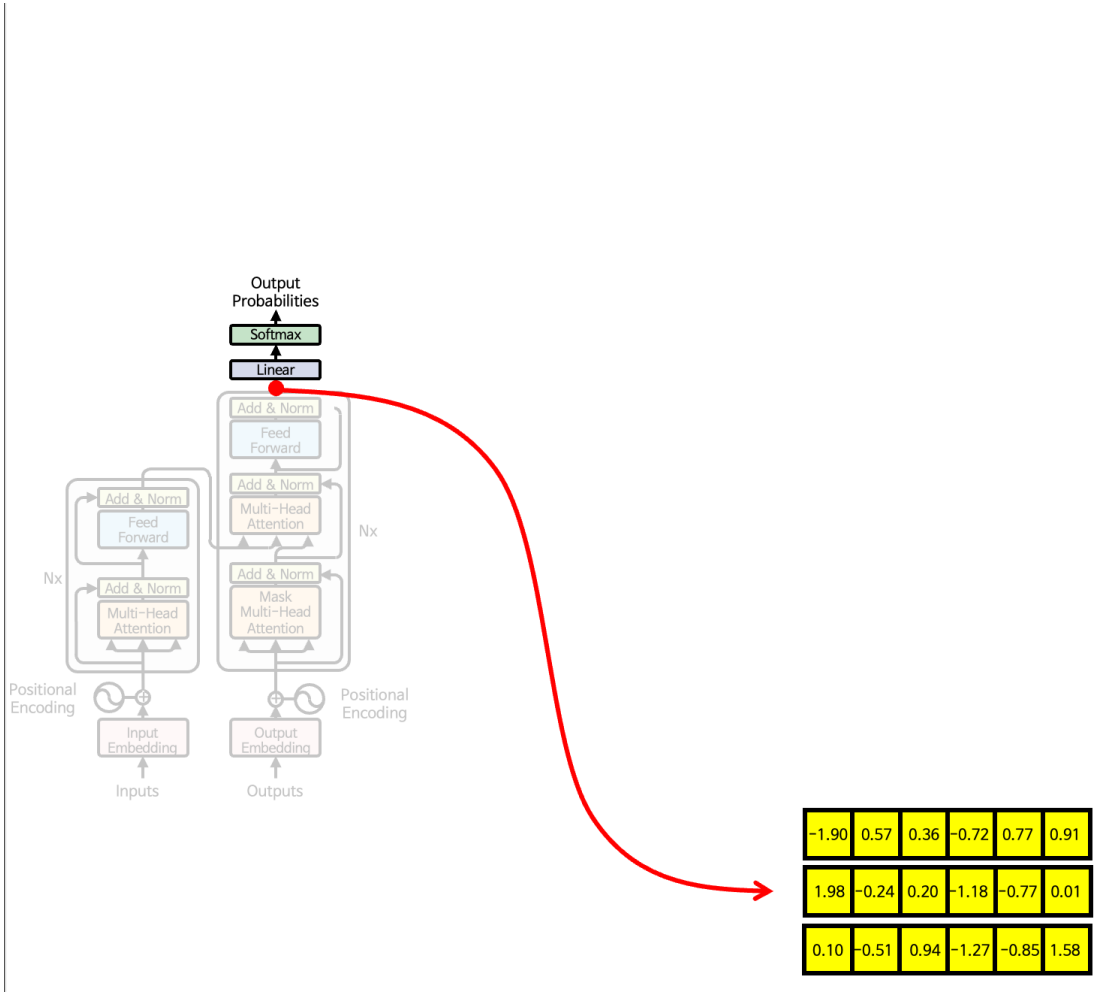
The same computational process is repeated, so details are omitted here.



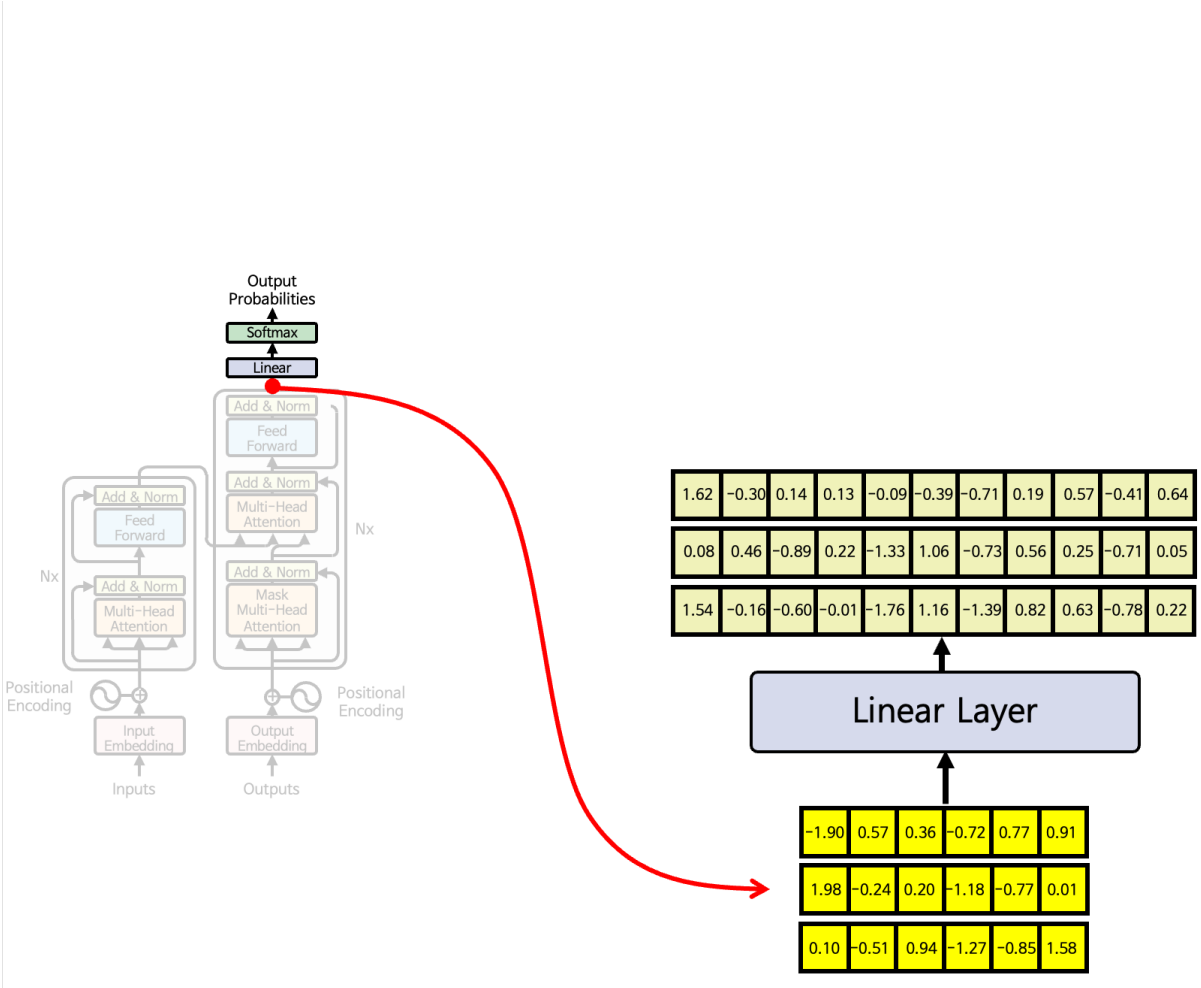
(5) The last linear and softmax layers produce the final output probabilities.



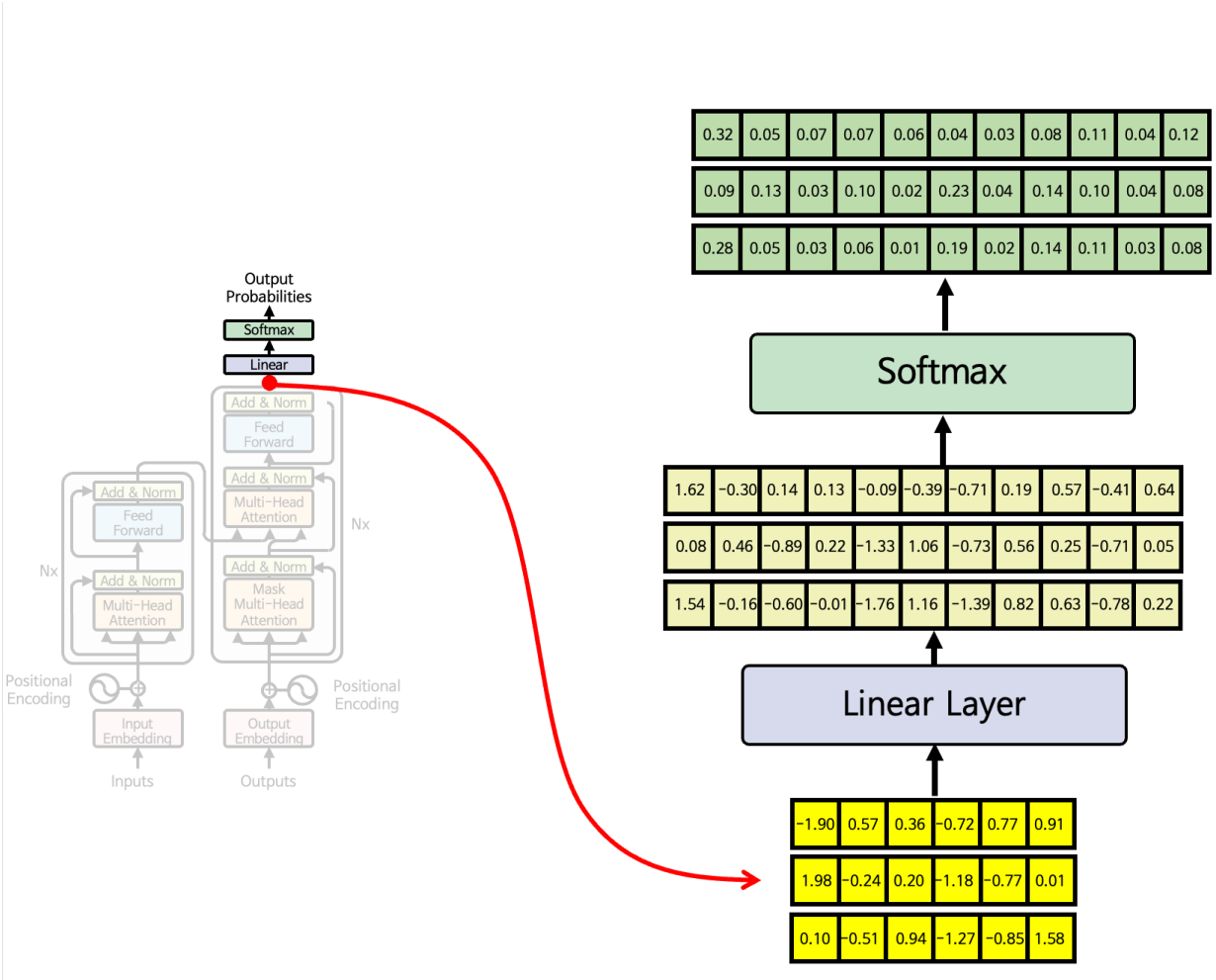
If the decoder output matrix at this stage looks like this:



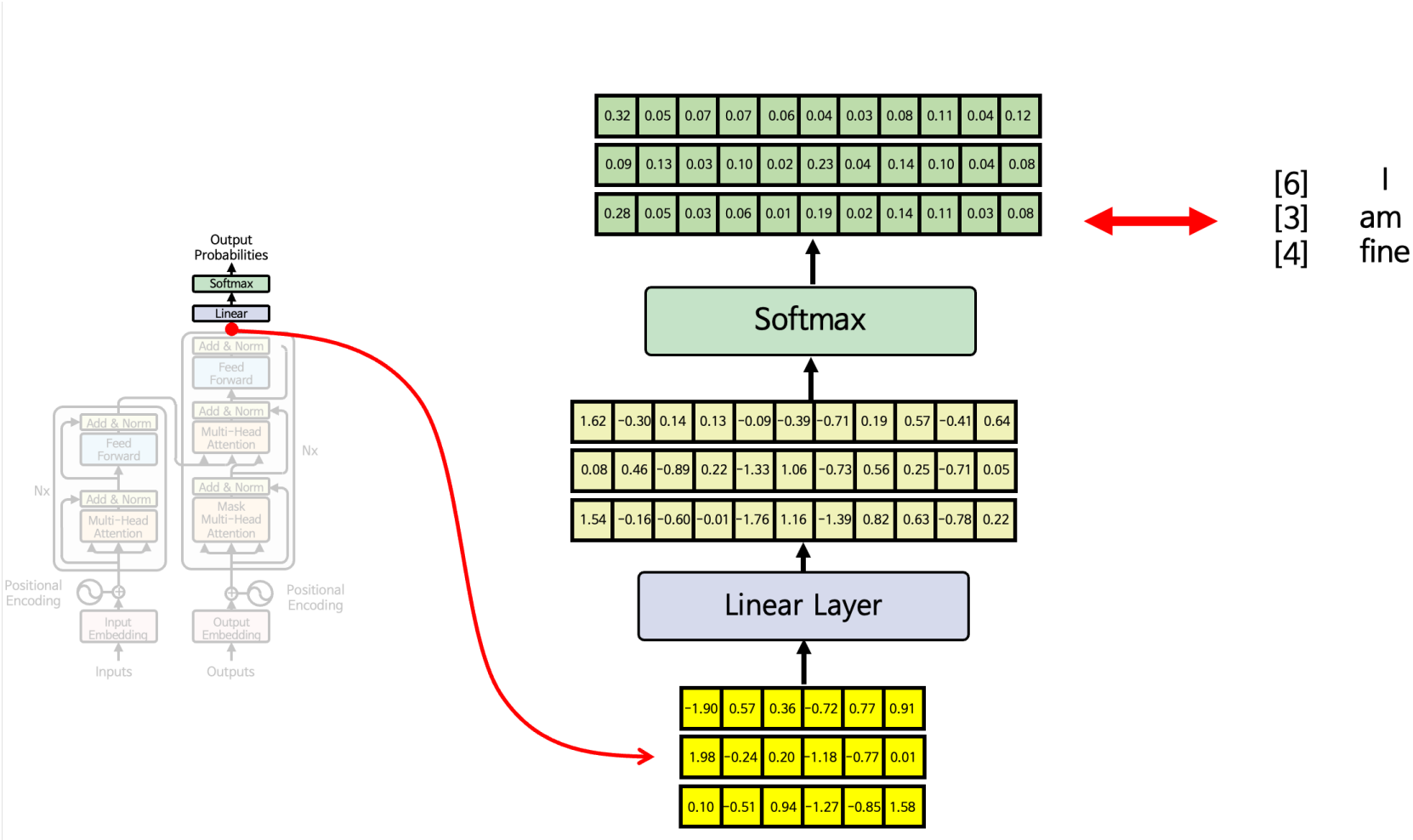
The linear layer projects it back to the full vocabulary size (=11).



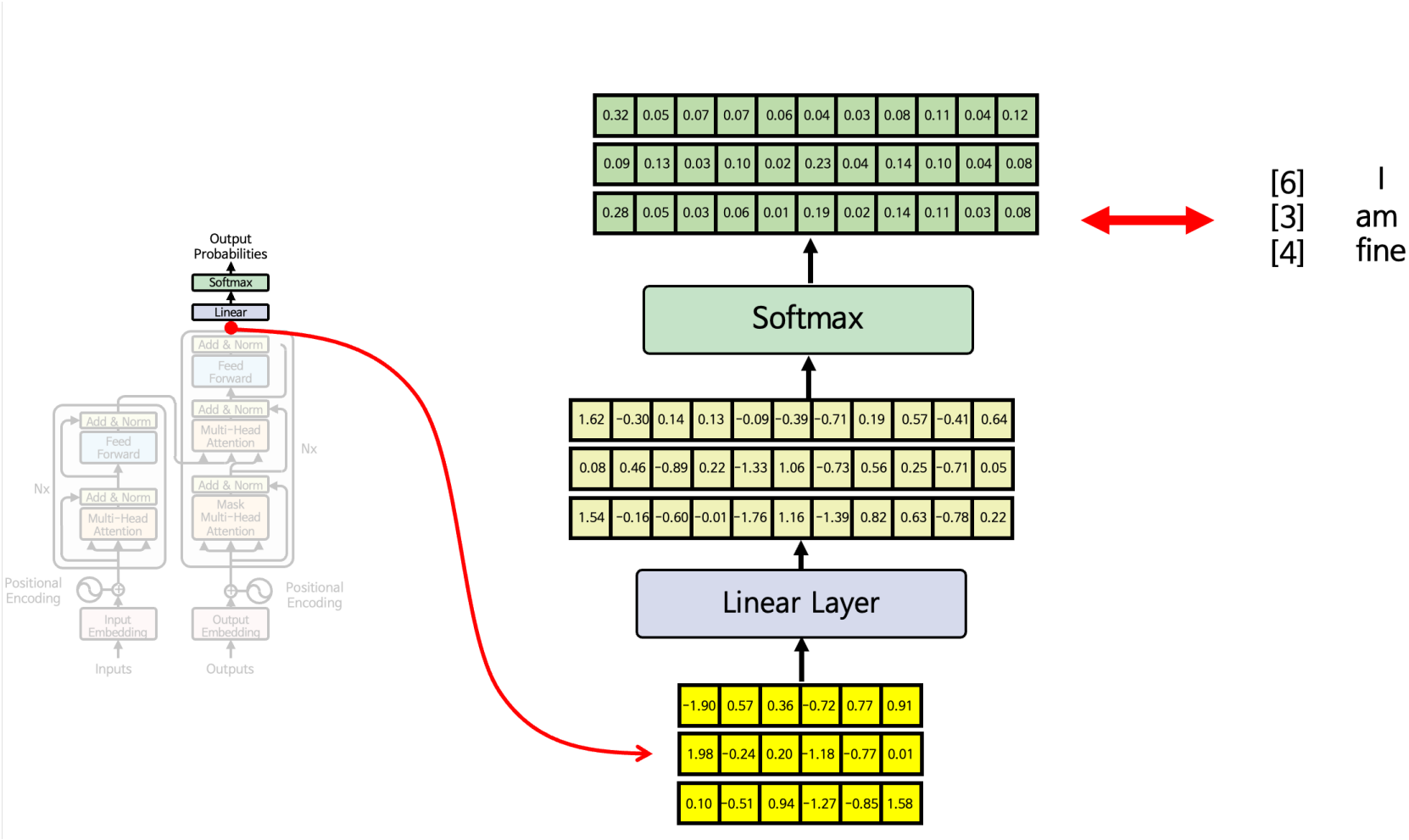
The softmax function then produces the final output probabilities.

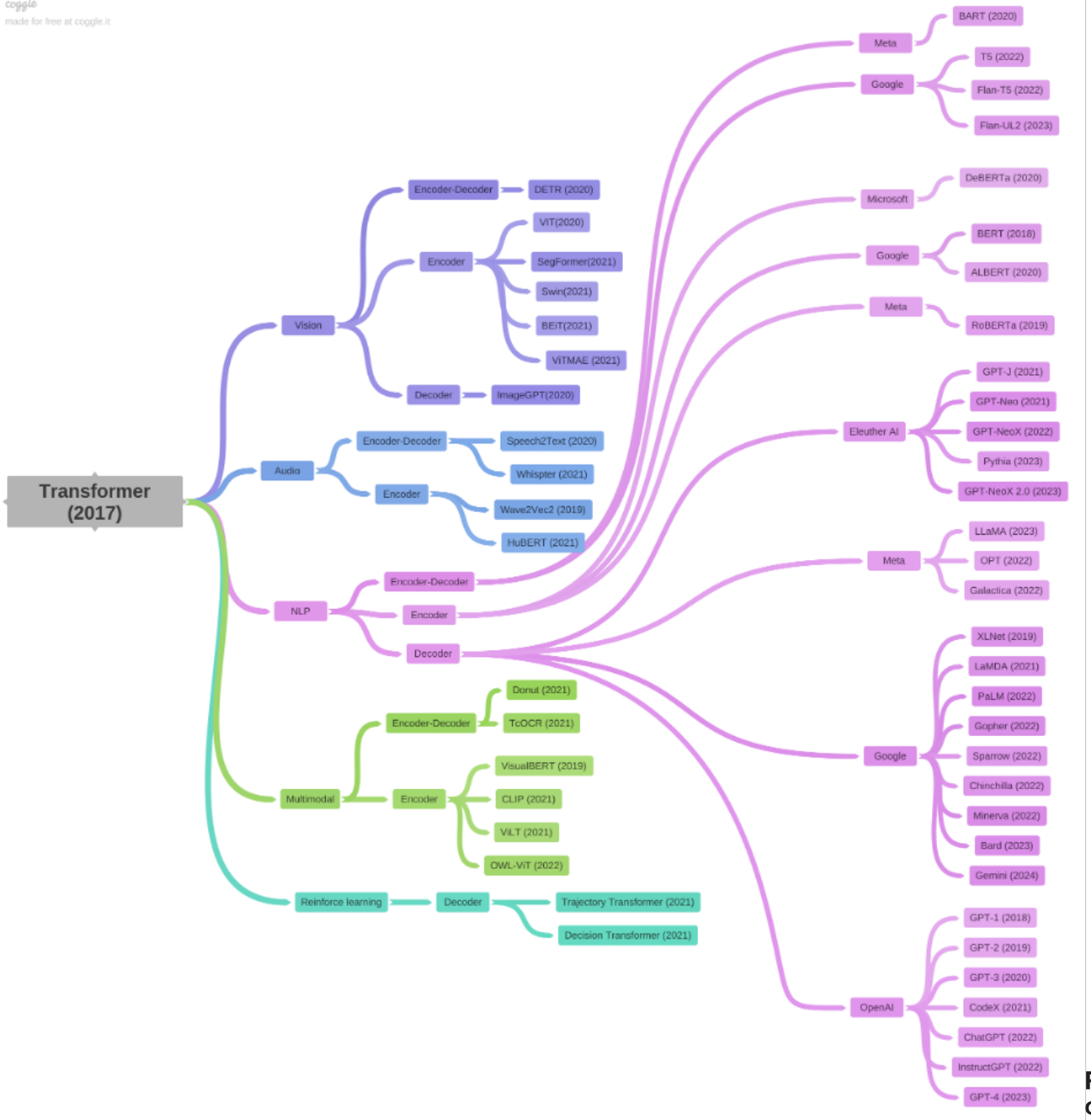


Finally, the predicted outputs are compared with the ground-truth labels (e.g., 6, 3, 4).



Using a **loss function** (e.g., cross-entropy) and **backpropagation**, the model updates all weight parameters across every layer — this is the learning process of the Transformer.





Presentation

Pooja - Devlin et al. (2019). *BERT: Pre-training of Deep Bidirectional Transformers*.

Outline

1 Attention

2 Transformer

3 Preview

4 Next Week & Upcoming Assignment

On Thursday

- Presentation: (1) Billy - Huang et al. (2018). *Music Transformer*; (2) Suruthi - Smith (2020). *Contextual Word Representation: A Contextual Introduction*
- Work on Lab 7 - Ollama

Outline

1 Attention

2 Transformer

3 Preview

4 Next Week & Upcoming Assignment

Next Week

Tuesday

- Project group meeting (Work in a group, Check-in with me)

Thursday

- Project proposal presentations (13 groups total; `https://hksung.github.io/Spring26_PSYC681/project/`)
- 5-minute informal update on your progress so far

Upcoming Deadline

- Submit the full project proposal by 3/13
- Guidelines: `https://hksung.github.io/Spring26_PSYC681/assignments/2_project%20proposal`